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Three-Position Temperature Control of a Nonlinear Process Model with Time Delay

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Abstract. In this paper, nonlinear automatic control systems are analyzed. Although there is a division into linear and nonlinear systems, complex systems that contain linear and nonlinear control elements are dealt with here. For the analysis of both, analysis and synthesis methods have been developed that are used in design. Analytical, graphical and grapho-analytical methods are used to check the stability of the management system. The methods usually use elementary operations from management and mathematics that have been developed for these needs. One system was chosen for temperature regulation, where elements were chosen whose mathematical representation complicates the system to the extent that it is difficult to analyze it using elementary operations. This paper applies the descriptive the descriptive function method that was developed for nonlinear systems. The goal will be achieved if, even in more complex algorithms for application, the result is obtained as efficiently as in simpler systems. For this, the apparatus of numerical mathematics will be used, for which a special calculation software has been developed.

Key-words: Control; descriptive function; numerical mathematics; software.

1. Introduction

This paper considers a system that contains a relay as a nonlinear element, and is therefore a system of nonlinear character. For the possibility of testing the system, harmonic linearization will be performed, which represents the nonlinear element with a descriptive function. Oscillatory modes will be examined, as the main property that follows nonlinear systems, i.e. self and forced oscillations. The configuration of the thermal system is such that, despite the harmonic linearization, it is impossible to find an analytical solution for the parameters of the oscillatory regimes. That is why the system will be solved by already known graphical methods, which are unsuitable both from the aspect of accuracy of results and time.

Papers in the management theory are are presented in the literature [1–4]. The papers [5–10] represent a group of papers from system identification. The field of numerical mathematics given by [11, 12]. Theory and practice from systems management is studied in [13–16]. References [17–20] are used for mathematical innovation of this paper. The paper [21] deals with the introduction of feedback and an algorithm for managing the crane system. The paper [22] deals with the application of phased systems in terms of controllers for integral processes. Through paper [23], the authors deal with the automated control of circuits using predictive control. The paper [24] describes the management and control of a nonlinear servo system using the Q learning algorithm. Applications arise from the presentation of theoretical contributions. The paper [25] contributes to the application of the PI phase controller where stability is evaluated. The applied research is finalized based on the theory. The references and experiences of the author of this work given in [27–35] also helped. Through the paper [36], a test platform with a graphical interface for drone control is presented. The paper [37] deals with a nonlinear servo system. The paper [38] illustrates the optimization of learning-based control on a crane system. Through paper [39], the authors analyze the optimal control of a levitation-type train. The paper [40] uses neural networks to control an experimental servo system. The paper [41] describes a hybrid control system for the crane executive body.

Te basic characteristics of nonlinear systems will be given in the next section of this paper. After the introductory remarks, an example of a system for automatic temperature regulation will be presented. The paper projects and analyzes the stability of a nonlinear system. Numerical methods are used with the use of a computer. A simulation package is also used for the purpose of checking. Due to the length of this paper, a part of the results is posted at [42]. The method described in this paper shows that it is possible to analyze more complex nonlinear systems.

2. Description of Nonlinear Systems and Functional Scheme of the Control System

A control system is said to be nonlinear if it has a feedback loop and contains at least one element whose dynamic functioning is described by a nonlinear relation. Nonlinear elements are relay elements, elements in segments with a linear characteristic, logic type elements, elements with an insensitivity zone, etc. The main characteristic of these elements is that the function is described by a nonlinear characteristic. Nonlinearity is divided into static and dynamic, which respectively correspond to nonlinear static characteristics.

Those characteristics are described by nonlinear differential equations. The main difference between nonlinear and linear systems is that the principle of superposition does not apply to nonlinear systems. The principle of superposition in linear systems has the property that the action of simultaneous inputs it can be represented as a sum of individual inputs. If the linear system is derived from the initial state, it can oscillate at frequencies determined by the roots of the characteristic equation of the real part.

The system excited by a certain frequency can oscillate only at that frequency. With nonlinear systems in the regime of forced oscillations, some parts of the system may oscillate at frequencies that are different from the input. The definitions of subharmonic and supraharmonic oscillations follow from there. If a nonlinear system can be described by a relation

$$F_1\left[x^{(n)}, x^{(n-1)}, \ldots, x, t\right] = 0 \tag{1}$$

then it is said to be a non-autonomous system. For the shape system

$$F_2\left[x^{(n)}, x^{(n-1)}, \ldots, x\right] = 0 \tag{2}$$

is said to be an autonomous system. A differential equation is usually written in the form

$$F_1\left[x^{(n)}, x^{(n-1)}, \ldots, x\right] = f(t). \tag{3}$$

If $f(t) = A\sin\omega t$, then the dynamic system is in the mode of forced oscillations. If the input is zero, such a system is in the mode of self-oscillations. The process of modeling nonlinear systems is performed by first linearizing the equations that describe the nonlinear elements. In this case, the nonlinearities are approximated by linear models.

Since this paper deals with a system for automatic temperature regulation, it contains a nonlinear element. The system has a magnetic amplifier, a polarized relay as a nonlinear element and other necessary elements for the operation of the system. The process should have a constant temperature that represents the output of the system. The process is controlled by turning on a relay as needed to activate the heater or cooling system. The temperature sensor is a thermocouple or a thermistor connected to the bridge.

The change in temperature is converted through the sensor and the bridge into a voltage that is sent to the magnetic amplifier as a control, where T is the pure delay. The output of the amplifier controls the switching on of the heating or cooling relay. A simplified transmission of a magnetic amplifier can be represented by the relation

$$W(s) = \frac{U_0(s)}{U_p(s)} = \frac{k}{Ts+1}, \tag{4}$$

where k represents the gain of the magnetic amplifier. That gain is determined by the slope of the static characteristic part. The relay element is of the polarized type where x is the input variable and y is the output of the relay. A real relay has hysteresis in real conditions. The thermal process has a pure delay and the transmission can be represented by

$$W_3(s) = \frac{e^{-Ts}}{1 + T_3 s}, \tag{5}$$

where T is the pure delay and T_3 is the time constant of the process. For a temperature control system with a relay and a delay, the thermodynamic equilibrium equation can be written in the time domain

$$\rho C V \frac{d\theta}{dt} = Q_g - hA(\theta - \theta_0). \tag{6}$$

Dividing (6) by hA gives

$$\frac{\rho C V}{hA}\frac{d\theta}{dt} + \theta = \frac{Q_g}{hA} + \theta_0. \tag{7}$$

Finding the Laplace transform from expression (7) gives

$$\frac{\rho CV}{hA}s\theta(s)+\theta(s)-\theta(0)=\frac{Q_g}{hA}+\theta_0. \tag{8}$$

The values specifed in (6) to (8) can be found in the specific example from the work and in the computer program from the attachment at the link [42].
By introducing exchanges

$$T_3=\frac{\rho CV}{hA},\quad \theta_g=\frac{Q_g}{hA},\quad \text{and}\quad \theta_s=\theta_0+\theta(0) \tag{9}$$

and by solving (8) by $\theta(s)$ we get the transfer at the regulated temperature in the form

$$\theta_s=\frac{\theta_g+\theta_s}{T_3s+1}, \tag{10}$$

where θ_g is the temperature of the heater, θ_s is the external temperature and θ is the temperature of the process. Considering the previous equalities, a block diagram of the control system can be formed, which is given in Fig. 1.

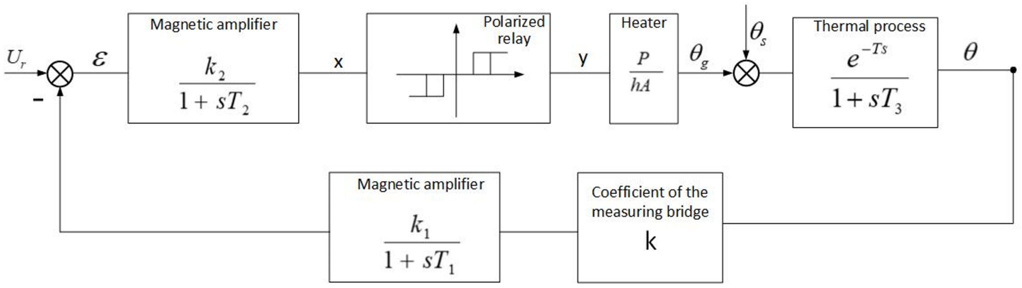

Fig. 1. Block diagram of the nonlinear control system

Depending on the voltage value, when the output is positive, the heater is turned on. At a negative value, the fan for cooling the regulated space is turned on. The part where the delay is marked represents the block where the temperature is regulated. The system has a feedback loop for the accuracy of temperature regulation.

Although there are now much more modern components for the realization of such systems, here the request is made to obtain a complex nonlinear system. The thus obtained system will be tried to be analyzed using classical analysis methods. The paper asks whether it is possible to solve and analyze such a system using elementary methods that are applied to simpler systems?

3. Introduction of Harmonic Linearization

A number of approximation methods have been developed for the analysis of nonlinear systems [1–4]. One such method is the harmonic linearization method. That method is based on the assumption that in the observed system a periodic movement regime is established on the unpaired with unknown amplitude A and oscillation frequency ω. This regime can also occur in systems without feedback but with external periodic excitations.

Self-oscillation mode can be established in nonlinear systems with feedback and without the effect of external excitation. This mode is characterized by the fact that all variables in the system are functions of time and as such can be developed into the Fourier series. Any nonlinear system can be represented by a block representing the linearized part L and the nonlinear one as F. The input to the nonlinearity is x and the output is y.

Elementary theory in the field of approximations of nonlinear systems is given in Attachment 1 [42]. The expression

$$N(A,\omega) = N_1(A,\omega) + \frac{N_2(A,\omega)}{\omega} s \tag{11}$$

it is called the descriptive function of the nonlinearity. These coefficients can be calculated for each of the nonlinearities that are characteristic.

Using Fig. 2, starting from the differential equations that describe the dynamic behavior of the linear part of the system, it is possible to form the equation

$$Q(s)x = -P(s)y, \tag{12}$$

where $Q(s)$ and $P(s)$ are polynomials by the differentiation operator s. With zero initial conditions, the transfer function of the linear part of the system given as $P(s)/Q(s)$ can be formed. We call such a function the equivalent linear part. If we proceed from (12) with relation (11), it is possible to form a general differential equation of the form

$$\left[Q(s) + P(s)\left(N_1(A,\omega) + \frac{N_2(A,\omega)}{\omega s}\right)\right] x = 0, \tag{13}$$

where $s \equiv d/dt$ is the differentiation operator.

For the mode of self-oscillations, the amplitude A and the frequency ω are constant values. Then, the periodic solutions, with $x \neq 0$, are obtained from the roots of the characteristic equation

$$\left[Q(s) + P(s)\left(N_1(A,\omega) + \frac{N_2(A,\omega)}{\omega s}\right)\right] = 0. \tag{14}$$

To find the mentioned parameters, the Mihailov curve can be used, so that the polynomial (14) is used, where for $s = j\omega$, the characteristic equation is transformed into

$$f(j\omega^*) = \left[Q(j\omega^*) + P(j\omega^*)\left(N_1(A,\omega) + \frac{N_2(A,\omega)}{\omega} j\omega^*\right)\right], \tag{15}$$

where ω^* is a current parameter and which should not be equated with the required frequency of self-oscillations ω. Separating the real and imaginary parts of (15) results in

$$f(j\omega^*) = R(A,\omega,\omega^*) + jI(A,\omega,\omega^*). \tag{16}$$

In order for (14) to have purely imaginary solutions, the Mihailov curve must pass through the coordinate origin, where the frequency of self-oscillations ω is determined for the value ω^* in the coordinate origin. This means that the following condition must be met:

$$R(A,\omega) = 0 \quad \text{and} \quad I(A,\omega) = 0 \tag{17}$$

by means of which the parameters of self-oscillations are found. In the case that A and ω are positive, this means that a mode of self-oscillations has arisen. Analytically, it is often difficult to find solutions, so one resorts to finding values graphically. In this case, since it is a deliberately chosen complex nonlinear system, numerical methods with computer programming will be used to solve it.

4. Analysis and Practical Realization of the Thermal System and Discussion

4.1. Theoretical derivation of system equations and adaptation for practical calculation

The temperature control system shown in Fig. 2 can be simplified by separating the linear equivalent part (L) from the nonlinear part (F). In order to do that, it is necessary to introduce the following relations:

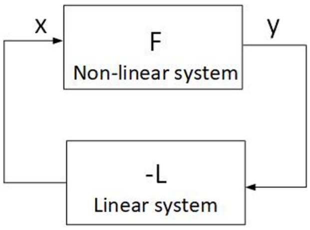

Fig. 2. Block diagram with separate linear and nonlinear parts of the system

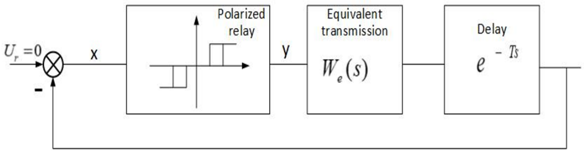

Fig. 3. Block diagram of the reduced control scheme

$$W_1(s) = \frac{k_1}{1 + sT_1}, \tag{18}$$

$$W_2(s) = \frac{k_2}{1 + sT_2}, \tag{19}$$

$$\theta(s) = \frac{\theta_g + \theta_s}{T_3 s + 1}. \tag{20}$$

The following relationships results from Fig. 2 and Fig. 3:

$$E = U_r - kW_1(s)\theta, \tag{21}$$

$$Q_g = Py, \quad \theta_g = \frac{Py}{hA}, \tag{22}$$

$$\theta = \frac{\theta_g + \theta_s}{1 + sT_3} e^{-Ts}, \tag{23}$$

$$E = U_r - kW_1(s)W_3(\theta_g + \theta_s). \tag{24}$$

Since it is valid

$$x = W_2(s)E, \tag{25}$$

then by substituting (25) into (24) and taking into account (18) to (23), with $U_r = \theta_s = 0$, the relation is obtained

$$-\frac{x}{y} = \frac{k_e e^{-Ts}}{(1+sT_1)(1+sT_2)(1+sT_3)}, \tag{26}$$

where k_e is the equivalent gain given by the relation

$$k_e = \frac{kk_1k_2}{hA}P. \tag{27}$$

The transmission P is given as

$$P = \frac{P_g}{U}, \tag{28}$$

where P_g is the power of the heater and U is the voltage with which the relay supplies the heater. Analogous to equation (26), the block diagram of the control system can be given in Fig. 3. In Fig. 3, W_e represents the equivalent transmission given by the relation

$$W_e(s) = \frac{k_e}{(1+sT_1)(1+sT_2)(1+sT_3)} \tag{29}$$

and the block e^{-Ts} represents the pure system delay.

System example: The following values are adopted for the analyzed system: $T_1 = 1s$, $T_2 = 0.1s$, $T_3 = 6s$, $T = 0.2s$, $P_g = 10kW$, $U = 220V$, $b_1 = 1V$, $b_2 = 0.2V$, $k_1 = 500V/V$ and $k_2 = 20V/V$. From reference [1], the harmonic linearization coefficients for the relay are

$$N_1 = \frac{f}{A^2}\left(\sqrt{4A^2 - c} + \sqrt{4A^2 - d}\right), \quad N_2 = \frac{g}{A^2}, \tag{30}$$

where are they $c = b_1^2$, $d = b_2^2$, $f = U/3.14159$ and $g = U(b_2 - b_1)/3.14159$.

By introducing shifts

$$\begin{aligned} T_a &= T_1 + T_2 + T_3, \\ T_b &= T_1T_3 + T_1T_2 + T_2T_3, \\ T_c &= T_1T_2T_3, \end{aligned} \tag{31}$$

where are they bearing in mind (29) as for the system from Figure 3, the characteristic equation, for $s = j\omega$, will have the form

$$1 + j\omega T_a - T_b\omega^2 - jT_c\omega^3 + k_e(N_1 + jN_2)e^{-j\omega T} = 0. \tag{32}$$

Let us arrange the previous equation in the form, with a known relation $e^{jx} = \cos x + j\sin x$,

$$\frac{\cos\omega T + j\sin\omega T}{k_e}\left[1 - T_b\omega^2 + j(T_a - T_c\omega^2)\omega\right] = -(N_1 + jN_2). \tag{33}$$

By arranging (33), the real part on the left side and the imaginary part on the right side of the equation are shown. The left side represents the reciprocal value of $W_e e^{-Ts}$ and the right side the equivalent nonlinearity defined through the descriptive function taken with a negative value. If we draw a separate diagram for the left and right sides (33) taking the abscissa axis as the real part and the ordinate axis as the imaginary one, two curves will be obtained. When those two curves intersect, the system will have its own oscillation.

It is shown in Attachment 2 [42] that for the previously given system parameters, the intersection is determined for $A = A_s = 1.2V$ and $\omega = \omega_s = 1.4\,\frac{rad}{s}$. Curves a, b and c are drawn in the mentioned figure for different parameters of the relay. Curves are also drawn for various values of the delay T. Each intersection of the curves (drawn with dashed and solid lines) gives the parameters of its own oscillations. Curve c does not have any intercepts, which proves that there are no self-oscillations. As is known, since the curve $(W_e(s)e^{-Ts})^{-1}$ covers the entire curve $-N(A)$, it proves that the system is stable.

4.2. Numerical calculation of oscillation parameters

Since the method of harmonic linearization when solving the system already introduced a certain error in the solution, the graphical solution of natural and forced oscillations introduces a new added error, which can make the result unreliable. The system analyzed in this paper is analytically unsolvable in terms of finding the amplitude and frequency of oscillation. The graphic solution mentioned above gave rough results, and in addition, it requires the tabulation of functions and their drawing. It is suggested that by means of a digital computer it is possible to draw these curves and so determine the solution in the section. However, as will be shown later, it is better to numerically calculate the parameters of the system with maximum accuracy.

This method will also solve the problem of system synthesis, which will show the universality of the procedure. In the literature, where there is no possibility of analytical solution, the graphic solution procedure is generally mentioned. Therefore, in this work, an attempt is made to break out of conventional frameworks, which will give the work a new approach. The procedure of iterative calculation will be chosen because it converges quickly and the solution is found with few iterations. If (30) is replaced in (33) and by arranging and equating the real and imaginary parts to zero, the following system is obtained:

$$(1 - T_b\omega^2)\cos\omega T - \omega(T_a - T_c\omega^2)\sin\omega T = -\frac{k_e f}{A^2}\left(\sqrt{4A^2 - c} + \sqrt{4A^2 - d}\right),$$

$$\omega(T_a - T_c\omega^2)\cos\omega T + (1 - T_b\omega^2)\sin\omega T = -\frac{k_e g}{A^2}, \tag{34}$$

where ω and A are unknowns. Since system (34) is nonlinear to solve, a numerical iterative method for solving it will be presented briefly.

Let's look at the system of equations

$$f_1(x,y) = 0 \quad \text{and} \quad f_2(x,y) = 0. \tag{35}$$

According to references [18, 19] for the system of equations (35) the iteration procedure can be established and that

$$x_{n+1} = x_n - \frac{1}{J(x_n,y_n)}\left[f_1(x_n,y_n)f'_{2y}(x_n,y_n) - f_2(x_n,y_n)f'_{1y}(x_n,y_n)\right], \tag{36}$$

$$y_{n+1} = y_n - \frac{1}{J(x_n,y_n)}\left[f_2(x_n,y_n)f'_{1x}(x_n,y_n) - f_1(x_n,y_n)f'_{2x}(x_n,y_n)\right],$$

where $(n = 0, 1, 2, \ldots)$ is the number of iterations. The Jacobian is given by the relation

$$J(x,y) = \frac{\partial f_1}{\partial x}\frac{\partial f_2}{\partial y} - \frac{\partial f_1}{\partial y}\frac{\partial f_2}{\partial x} \tag{37}$$

We will apply this iterative calculation procedure to the calculation of the parameters of the self-oscillations of the system for automatic temperature regulation. Let us form the following functions

$$\begin{aligned} f_1 &\equiv (1 - T_b\omega^2)\cos\omega T - \omega(T_a - T_c\omega^2)\sin\omega T + \frac{k_e f}{A^2}\left(\sqrt{4A^2 - c} + \sqrt{4A^2 - d}\right), \\ f_2 &\equiv \omega(T_a - T_c\omega^2)\cos\omega T + (1 - T_b\omega^2)\sin\omega T + \frac{k_e g}{A^2}, \end{aligned} \tag{38}$$

where f_1 is the real part and f_2 is the imaginary part of the characteristic equation. It can be seen from (38) that it is impossible to analytically explicitly express the amplitude and frequency of self-oscillations for the reason that a nonlinear system needs to be solved. Therefore, we will apply to (38) the iterative solution procedure given by relations (36) and (37). Let us find the derivatives of the functions f_1 and f_2 which are given by the following relation

$$f'_{1\omega} = -2T_b\omega\cos\omega T - T(1 - T_b\omega^2)\sin\omega T - (T_a - 3T_c\omega^2)\sin\omega T - T(\omega T_a - T_c\omega^3)\cos\omega T, \tag{39}$$

$$f'_{2\omega} = (T_a - 3T_c\omega^2)\cos\omega T - T(\omega T_a - T_c\omega^3)\sin\omega T + T(1 - T_b\omega^2)\cos\omega T - 2T_b\omega\sin\omega T,$$

$$f'_{1A} = -\frac{2k_e f}{A^3}\left(\sqrt{4A^2 - c} + \sqrt{4A^2 - d} - \frac{2A^2}{\sqrt{4A^2 - c}} - \frac{2A^2}{\sqrt{4A^2 - d}}\right), \quad f'_{2A} = -\frac{2k_e g}{A^3}, \tag{40}$$

thus the iterative procedure is fully determined. Based on relations (38) and (39), programming was performed in one of the higher programming languages for a digital computer in [42], which is given in the attachment of this work. The program is intended for numerical calculation of the self-oscillations of the system designed in this work. The procedure is iterative and since it converges quickly, the result is obtained in several iterations.

Several examples were made where the parameters of the relay and the net delay of the system were changed. If the system has no solution, i.e. there are no self-oscillations, the program is coded that way because it is ensured that the corresponding message will be printed. Then the next data for processing is loaded, if there is any, otherwise the program ends. With a small change, only in the part of the function command, the program can be applied for any system configuration.

The modes of self-oscillations were found using the graphical method for several examples. By checking on a computer program, those results were confirmed, in the sense of whether or not there are self-oscillations. In addition, numerical values of oscillations were calculated for amplitude and frequency. Additionally, by testing the Jacobian value, the program finds whether its own oscillations are stable or not. The program enables the behavior of the regulation system to be modeled by changing the parameter values according to the wishes and needs of the user.

5. Conclusions

A nonlinear system for automatic temperature regulation was analyzed using the harmonic linearization method. Primarily, it was shown that there is a possibility of analysis of such systems by applying the graphical method. Since this method is unsuitable for finding and solving the oscillatory mode, where it is necessary to calculate the amplitude and frequency of own oscillations as well as the measure of stability, the idea was created to use iterative methods of numerical mathematics. The system is designed from a polarized relay, which is a highly nonlinear element. The task is to design such a system for automatic temperature regulation of a complex structure. This complexity makes it difficult to apply descriptive function methods and creates distrust as to whether they will give positive results for analysis, since it is an approximate method.

Solving the system of nonlinear equations was done by choosing and adjusting the method of numerical mathematics. In addition to adapting this method, the solution was successfully implemented by programming in a higher programming language, which created a software package.The computer program for determining the required quantities and analyzing the stability of self-oscillations is given in Attachment 3 [42]. The software calculates the value of the amplitude and frequency of self-oscillations quickly and with high accuracy for given input values. In addition, it determined whether the system is stable or not, as well as the evaluation of the filtering characteristics of the system. It can be considered a certain result of the work that it has been shown that even more complex systems can be solved using the descriptive function method and that the results are reliable. By directly calculating some values for the selected example and entering them into the computer program, all the data were obtained. The experience in practical implementations in the production of two-position and three-position temperature regulators that have been functioning for a long time was used.

In order to verify the software package for solving nonlinear control systems, a program for modeling continuous systems (known as CSMP) was used Attachment 4 [42] (contains simulation program). The results obtained on this analog-digital simulator confirmed the results obtained in this work. It supports the claim that it provides users with a valuable method for system analysis and synthesis of more complex nonlinear systems. Since the computer programs for the calculation of many relevant quantities were made, the limit modes of the oscillatory process were determined. For the sake of clarification, by changing quantities such as the equivalent gain k_e, delay T, relay hysteresis and time constant T_3, the limit mode of self-oscillations is defined. It has been shown that systems in the regime of stable self-oscillations can also be derived from that regime. This can be of great use for practical applications.

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