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Modified Evolved Bat Algorithm of Fuzzy Optimal Control for Complex Nonlinear Systems

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Abstract. This paper proposes a novel artificial intelligence based Evolved Bat Algorithm (EBA) controller with machine learning matched membership functions in a complex nonlinear system. The proposed affine transformed membership functions are adopted and stabilized and closed-loop performance criteria TS fuzzy systems are obtained through a new parametric linear matrix inequality technology rearranged by a capacity function member that fits with machine learning. The fuzzy systems are described based on the optimal fuzzy logic control (FLC) approaches. In addition, the concentration can be reconstructed using two different free weight tables using a decision separation technique without scaling parameters. Numerical examples confirm the superiority of this method. Furthermore, a stability criterion for the complex stability radius is proposed to guarantee the *D*-stability of discrete time-delay fuzzy systems in the presence of consequent parametric uncertainties.

Keywords: AI, EBA, optimization, nonlinear system.

1. Introduction

Since conventional controllers cannot track control details, artificial intelligence and control technologies can be integrated to achieve more important standards. Computer-based intelligence can include a variety of calculation methods that try to mimic human thinking processes or the practices discovered in nature in order to solve a given problem [4], [5]. For management frameworks, these techniques can provide valuable and improved implementations based on any prior information metrics [15], [16]. The extended use of computer-based intelligence and its confirmations are usually due to certain important attributes. Moreover, they are adaptable, so hybrid strategies can be quickly found, including at least two computer-based intelligence programs and conventional technology. Finally, they are universal, so one can imagine applying them for various types of contextual analyses and encouraging the making of continuous improvements. It has proven to be a good alternative to linear controllers and can be used as viable options for controlling non-linear and uncertain combinations. A self-arranged fuzzy controller naturally functions to perfect the underlying fuzzy principles. The configuration of the fuzzy logic controller (FLC) should allow adaptability to change the control, because the included framework are often puzzling so that the characteristics of the components become poor with the occurrence of nonlinearity that fluctuates over time. Thus, the usual control techniques rely on the assumption of a straight frame, and a nonlinear frame is usually linearized before use. In addition, most controllers are created based on the precise numerical model of the framework. Indeed, it is difficult to accurately display some frames with a comprehensive numerical model, which has inspired the enthusiasm for using fuzzy controllers. Another preferred position is that the fuzzy controller be customizable and that it can be effectively understood by human experts because the standard-based control information is semantic [14], [20].

In the controller design process, the linear matrix inequality (LMI) approach and fuzzy state space modeling can be utilized for practical applications [17]. This FLC system deals with a general class of nonlinear processes and a stable design is carried out based on this stability analysis theorem by using the Lyapunov's energy concept and the framework for each rule in this fuzzy controller [18]. Similarly, fuzzy systems are applied as alternate model to make the best green ratios period and traffic light under the cycle times. This fuzzy system could be a better choice for the optimization of traffic applications [19].

The pole assignment problem in the straight frame hypothesis discussed by many authors can be understood in different ways. However, considering the way in which parameter loopholes begin to constitute various sources, the area of the shaft will change and cannot be fixed. As such, in a reasonable application, all the axes of the universal frame can be placed in an ideal area instead of choosing precise tasks. One such specific and very region for the discrete systems is example of a disk $D(\alpha, r)$, as shown in Fig. 1. The transfer system polar cap to a special disk which was referred to as a problem position display D-pole. On the other hand, many physical fuzzy complex systems have inherent much delay factors that can be considered by a time delay in differential or difference equations. As the number of poles in the capelin cluster system become large due to the happens of time delays, the application of the time delay coefficient of the D-pole problem becomes much more complicated.

Fuzzy control has been employed in many kinds of industrial applications. One of the most and the first questions that must be discussed and answered in this case is the stability of the fuzzy system. In the current researches, Tanaka and Sugeno [1], Tanaka and Sano [2], Zhao et al. [3] and Wang et al. [4] etc. have proposed some useful stability techniques to check the asymptotic stability of Takagi-Sugeno $(\mathrm{T}\text{-}\mathrm{S})$ fuzzy systems. The methods presented in these papers [1-4] require to specific find out a common positive definite matrix solution P for the Lyapunov equations. However, this is very complicated and it is not easy to find a common P in a general way. Hence, the purpose of this paper is to provide a stability criterion in term of a complex stability radius, which does not require solving any Lyapunov equation or to specific find out a common and very positive definite matrix solution for the Lyapunov equations, to guarantee the $D(\alpha, r)$-stability of the discrete time-delay T-S fuzzy systems in the presence of consequent parametric uncertainties.

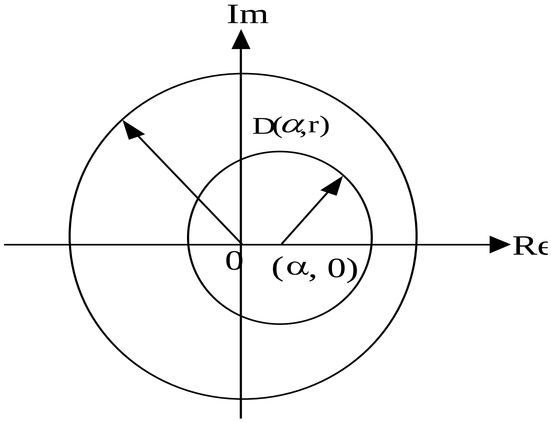

Fig. 1. A disk $D(\alpha, r)$ centered at $(\alpha, 0)$ with radius *r*

2. Preliminary

To proceed with the derivation of the *D*-stability criterion, some preliminary results are first introduced. All assumptions of Section 2 will clearly show the area of process control.

Definition 2.1: A dynamic $D(\alpha, r)$-stable has all poles within the specific disk $D(\alpha, r)$ centered (α, 0) in *r*, then the characteristic solutions satisfy $|(z-\alpha)/r|<1$ where $|\alpha|+r<1$ and $r>0$.

Definition 2.2 [5, 6]: Given all matrix *A* eigenvalues inside the unit circle, then

$$\rho(A) \equiv \left\{ \begin{matrix} \max \\ 0 \le \theta \le 2\pi \end{matrix} \left\{ \left\| \left[e^{j\theta} I - A \right]^{-1} \right\| \right\} \right\}^{-1}$$

is said to be a complex stability radius.

Remark 2.1 [6]: $\rho(A)$ depends on norm choice, i.e. given Euclidean norm, then

$$\rho(A) = \min_{0 \le \theta \le 2\pi} \left\{ \underline{\sigma}[e^{j\theta} I - A] \right\},$$

in which $\underline{\sigma}(\cdot)$ indicates the minimal singular value of matrix $(\cdot)$.

Lemma 2.1 [6]: Given all matrix M eigenvalues inside the unit disk of the complex plane, all matrices $M+\Delta M$ eigenvalues are inside the unit disk if and only if $\|\Delta M\| < \rho(M)$.

Lemma 2.2 [7]: Given a matrix $E(z) \in \Re_{\infty}^{m\times n}$ with $\Re_{\infty}^{m\times n}$ indicating the set of $m \times n$ matrices with rational functions, then

$$\sup_{z\in\Omega}\|E(z)\| = \sup_{|z|\ge 1}\|E(z)\| = \sup_{\theta\in[0,2\pi]}\left\|E(e^{j\theta})\right\|, \qquad (2.1)$$

where $\Omega \equiv \left\{ z = re^{j\theta}, \theta \in [0, 2\pi], |r| \le 1 \right\}$.

The controller design approach for a dynamic fuzzy complex system is described in Section 3.

3. Stability of T-S Fuzzy Systems

Based on the concept of stability of previous D stable criterion, let us consider a discrete time-delay T-S fuzzy system with consequent parametric and factor uncertainties, which is depicted by the following fuzzy implications:

Rule ^{i} : IF x_1 is M_{i1} $\cdots$ and x_m is M_{im}

$$\text{THEN} \quad X^i(k+1) = (A_i + \Delta A_i) X(k) + \sum_{j=1}^{n} (A_{dij} + \Delta A_{dij}) X(k - d_j) \tag{3.1}$$

in which x_ℓ $(\ell = 1, 2, \cdots, m)$ denote state variables and $X^T(k) \equiv [x_1(k), x_2(k), \cdots, x_m(k)]$; i =1…w and w IF-THEN rules. $M_{i\ell}$ fuzzy sets, $X^i(k+1)$ output , A_i , and A_{dij} constant matrices are of appropriate dimensions; ΔA_i and ΔA_{dij} are referred to as the parametric uncertainties (existing in the consequent part of the i^{th} IF-THEN rule) with follows:

$$\| \Delta A_i \| \le \beta_i ,\ \| \Delta A_{dij} \| \le \eta_{ij} ,\ i = 1, 2, \ldots, w;\ j = 1 \ldots n, \tag{3.2}$$

where β_i and η_{ij} are given constants. For $X(k)$, the final output is

$$X(k+1) = \sum_{i=1}^{w} \mu_i \{ (A_i + \Delta A_i) X(k) + \sum_{j=1}^{n} (A_{dij} + \Delta A_{dij}) X(k - d_j) \} \Big/ \sum_{i=1}^{w} \mu_i$$

$$= \sum_{i=1}^{w} h_i \{ (A_i + \Delta A_i) X(k) + \sum_{j=1}^{n} (A_{dij} + \Delta A_{dij}) X(k - d_j) \}, \tag{3.3}$$

with $\mu_i \equiv \prod_{\ell=1}^{m} M_{i\ell}(x_\ell)$ and $M_{i\ell}(x_\ell)$ is the membership degree of x_ℓ in $M_{i\ell}$; $h_i \equiv \mu_i \Big/ \sum_{i=1}^{w} \mu_i$ and hence $\sum_{i=1}^{w} h_i = 1$. Eq. (3.3) is next transformed into

$$X(k+1) = AX(k) + \Delta AX(k) + \sum_{j=1}^{n} A_{dj} X(k - d_j) + \sum_{j=1}^{n} \Delta A_{dj} X(k - d_j), \tag{3.4}$$

where

$$A = \sum_{i=1}^{w} h_i A_i ,\ \Delta A = \sum_{i=1}^{w} h_i \Delta A_i ,\ A_{dj} = \sum_{i=1}^{w} h_i A_{dij} ,\ \Delta A_{dj} = \sum_{i=1}^{w} h_i \Delta A_{dij} . \tag{3.5}$$

According to (3.2) and (3.5), the following relations result:

$$\| \Delta A \| \le \beta \equiv \sum_{i=1}^{w} h_i \beta_i ,\ \| \Delta A_{dj} \| \le \eta_j \equiv \sum_{i=1}^{w} h_i \eta_{ij} . \tag{3.6}$$

***Theorem 3.1*: (I)** Given all A eigenvalues within the specific disk $D(\alpha, r)$, the system (3.4), or equivalently the system (3.3), is robustly $D(\alpha, r)$-stable with $|\alpha| < r$, if

$$\frac{1}{r}\left[\beta + \sum_{j=1}^{n} \left(\|A_{dj}\| + \eta_j \right) \left(r - |\alpha| \right)^{-d_j} \right] \equiv d_1 < d_s \equiv \rho\left[\frac{A - \alpha I}{r} \right]. \tag{3.7}$$

(II) If $d_1 \ge d_s$ and this function

$$h(g) \equiv \frac{1}{r}\left[\beta + \left\| \sum_{j=1}^{n} A_{dj} (rg + \alpha)^{-d_j} \right\| + \sum_{j=1}^{n} \eta_j \left(r - |\alpha| \right)^{-d_j} \right] \tag{3.8}$$

is not within $[d_s,\ d_1]$, where $|\alpha| < r$, and g is bounded $U_1 = \{ g \mid g = re^{j\theta}, \theta \in [0, 2\pi], 1 \le r \le d_{1r} \}$ with $d_{1r} = \| (A - \alpha I)/r \| + d_1$, then Eq.

(3.4) is robustly $D(\alpha, r)$ -stable.

Proof: **(I)** The sufficient necessary condition ensuring poles lie inside $D(\alpha, r)$ is that all characteristic solutions of

$$\det\left\{zI-\left[A+\Delta A+\sum_{j=1}^{n}\left(A_{dj}+\Delta A_{dj}\right)z^{-d_j}\right]\right\}=0 \tag{3.9}$$

satisfy the condition $|z-\alpha|<r$. Let $(z-\alpha)/r$ be g (i.e., $z=rg+\alpha$), then (3.9) becomes

$$\det\left\{gI-\left[\frac{A-\alpha I}{r}+\frac{1}{r}\left(\Delta A+\sum_{j=1}^{n}\left(A_{dj}+\Delta A_{dj}\right)(rg+\alpha)^{-d_j}\right)\right]\right\}=0. \tag{3.10}$$

If in the case of $|g|\geq 1$, we have $r-|\alpha|\leq|rg+\alpha|$. From (3.7), the following inequality is obtained:

$$\frac{1}{r}\left[\|\Delta A\|+\sum_{j=1}^{n}\left(\|A_{dj}\|+\|\Delta A_{dj}\|\right)|rg+\alpha|^{-d_j}\right]<\rho\left(\frac{A-\alpha I}{r}\right)\ \text{for } |g|\geq 1 \tag{3.11}$$

$$\Rightarrow\left\|\frac{1}{r}\left[\Delta A+\sum_{j=1}^{n}\left(A_{dj}+\Delta A_{dj}\right)(rg+\alpha)^{-d_j}\right]\right\|<\rho\left(\frac{A-\alpha I}{r}\right)\ \text{for } |g|\geq 1. \tag{3.12}$$

Due to all *A* eigenvalues are within $D(\alpha, r)$, all $(A-\alpha I)/r$ eigenvalues are in unit circle. Hence, based on (3.12) and Lemma 2.1, we have

$$\left|\lambda\left\{\frac{A-\alpha I}{r}+\frac{1}{r}\left[\Delta A+\sum_{j=1}^{n}\left(A_{dj}+\Delta A_{dj}\right)(rg+\alpha)^{-d_j}\right]\right\}\right|<1\ \text{for } |g|\geq 1. \tag{3.13}$$

This means that

$$|g|\neq\left|\lambda\left\{\frac{A-\alpha I}{r}+\frac{1}{r}\left[\Delta A+\sum_{j=1}^{n}\left(A_{dj}+\Delta A_{dj}\right)(rg+\alpha)^{-d_j}\right]\right\}\right|\ \text{for } |g|\geq 1. \tag{3.14}$$

In view of (3.14), all solutions of the characteristic equation (3.10) satisfy $|g|<1$, i.e., $|(z-\alpha)/r|<1$. This completes the proof of case (I).

(II) If Eq. (3.4) is not $D(\alpha, r)$ -stable, then a solution g of Eq. (3.10) satisfies

$$|g|=\left|\lambda\left\{\frac{A-\alpha I}{r}+\frac{1}{r}\left[\Delta A+\sum_{j=1}^{n}\left(A_{dj}+\Delta A_{dj}\right)(rg+\alpha)^{-d_j}\right]\right\}\right|\geq 1. \tag{3.15}$$

Based on Lemma 2.1 and (3.15), the following inequality is obtained:

$$\begin{aligned}
d_s=\rho\left(\frac{A-\alpha I}{r}\right)&\leq\left\|\frac{1}{r}\left[\Delta A+\sum_{j=1}^{n}\left(A_{dj}+\Delta A_{dj}\right)(rg+\alpha)^{-d_j}\right]\right\|\\
&\leq\frac{1}{r}\left[\beta+\left\|\sum_{j=1}^{n}A_{dj}(rg+\alpha)^{-d_j}\right\|+\sum_{j=1}^{n}\eta_j\,(r-|\alpha|)^{-d_j}\right]=h(g)\\
&\leq\frac{1}{r}\left[\beta+\sum_{j=1}^{n}\left(\|A_{dj}\|+\eta_j\right)\left(r-|\alpha|\right)^{-d_j}\right]=d_1\ \ \text{for } |g|\geq 1.
\end{aligned} \tag{3.16}$$

Moreover, according to (3.15), the following inequality is get:

$$1 \le |g| = \left| \lambda \left\{ \frac{A - \alpha I}{r} + \frac{1}{r} \left[\Delta A + \sum_{j=1}^{n} \left(A_{dj} + \Delta A_{dj} \right) (rg + \alpha)^{-d_j} \right] \right\} \right|$$
$$\le \left\| \frac{A - \alpha I}{r} \right\| + \frac{1}{r} \left[\beta + \sum_{j=1}^{n} \left(\| A_{dj} \| + \eta_j \right) (r - |\alpha|)^{-d_j} \right] = d_{1r} . \tag{3.17}$$

Therefore, given Eq. (3.4) not $D(\alpha, r)$ -stable, then all complex system unstable poles must be considered within the bounded region U_1. That is, given (3.16) not regarded with true (i.e., $h(g)$ does not lie inside in the interval $[d_s,\ d_1]$ for all $g \in U_1$), so Eq. (3.4) is really and robustly $D(\alpha, r)$ -stable. These completes the proof of case (II).

***Remark 3.1*:** Theorem 3.1 Case (I) derived an algebraic condition to assess error in the Eq. (3.4) D-stability with the cost of conservativeness. Then checking the D-stability in the case (I) and, if it was failed, it resorts to case (II). Thus, case (I) and case (II) complement between each other.

However, for practical purposes, it is difficult to consider a scenario proposal (II) 3.1. It may be "The marginal test" in checking this situation.

***Corollary 3.1*:** If $d_1 \ge d_s$ and the following condition holds:

$$h(g) = \frac{1}{r} \left[\beta + \left\| \sum_{j=1}^{n} A_{dj} (rg + \alpha)^{-d_j} \right\| + \sum_{j=1}^{n} \eta_j (r - |\alpha|)^{-d_j} \right] < d_s , \tag{3.18}$$

where $|\alpha| < r$ and $g = e^{j\theta}$ for $\theta \in [0, 2\pi]$, then Eq. (3.4) is robustly $D(\alpha, r)$ -stable.

Proof: The matrix $\sum_{j=1}^{n} A_{dj} (rg + \alpha)^{-d_j}$ of in which all poles with the modulus $|g| = |\alpha| / r < 1$ belong to $\Re_{\infty}^{m \times m}$. Consequently, on the basis of Lemma 2.2, the term $\| \sum_{j=1}^{n} A_{dj} (rg + \alpha)^{-d_j} \|$ in (3.18) takes on its supremum in the kind of range, which are given by $g = e^{j\theta}$ for $\theta \in [0, 2\pi]$. Therefore, if Eq. (3.18) holds, $h(g)$ does not been lie inside the interval $[d_s,\ d_1]$ for all $g \in U_1$, thus it is shown that Eq. (3.4) is robustly $D(\alpha, r)$ -stable (according to Theorem 3.1 case (II)). This completes the proof.

To ensure the stability in complex nonlinear, fuzzy models and stability analysis are indicated by T–S fuzzy models. The concept of PDC, which is described in the Fig. 2, helps to derive fuzzy control structures from T–S fuzzy models and i^{th} rule is considered in [9], [10]

Rule i : IF $x_1(t)$ is M_{i1} and $\cdots$ and $x_p(t)$ is M_{ip} (3.19)

THEN $\dot{\boldsymbol{X}}(t) = \boldsymbol{A}_i \boldsymbol{X}(t) + \boldsymbol{B}_i \boldsymbol{U}(t) + \boldsymbol{E}_i \boldsymbol{\varphi}(t)$, (3.20)

where $\boldsymbol{X}(t)$ is the state vector, M_{ip} ($p = 1,\ 2, \cdots, g$) are the fuzzy sets, $x_1(t) - x_p(t)$ are the premise variables, with appropriate singleton fuzzifier, the product inference, and the center average defuzzifier, and the dynamic Eq. (3.20) is

$$\dot{\boldsymbol{X}}(t) = \frac{\sum_{i=1}^{r} w_i(t) [\boldsymbol{A}_i \boldsymbol{X}(t) + \boldsymbol{B}_i \boldsymbol{U}(t) + \boldsymbol{E}_i \boldsymbol{\varphi}(t)]}{\sum_{i=1}^{r} w_i(t)} = \sum_{i=1}^{r} h_i(t) \left[\boldsymbol{A}_i \boldsymbol{X}(t) + \boldsymbol{B}_i \boldsymbol{U}(t) \right] + \boldsymbol{E}_i \boldsymbol{\varphi}(t) , \tag{3.21}$$

with

$$w_i(t) = \prod_{p=1}^{g} M_{ip}\left[x_p(t)\right],\quad h_i(t) = \frac{w_i(t)}{\sum_{i=1}^{r} w_i(t)}, \tag{3.22}$$

$M_{ip}\left[x_p(t)\right]$ is the membership degree of $x_p(t)$ in M_{ip} . It is regarded that

$$w_i(t) \geq 0,\quad i = 1, 2, \cdots, r;\quad \sum_{i=1}^{r} w_i(t) > 0 . \tag{3.23}$$

for all *t*. Therefore, $h_i(t) \geq 0$ and $\sum_{i=1}^{r} h_i(t) = 1$ for all *t*.

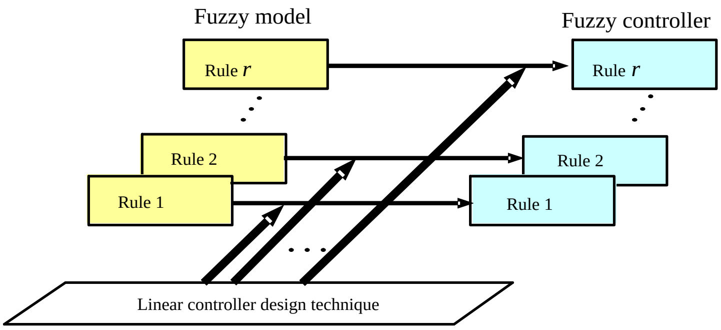

Fig. 2. Parallel Distributed Compensation (PDC) design

To design the universal control for fuzzy models (3.20), PDC is next applied in this paper. With the same criteria (3.20), the first rule of FLC can be taken [12], [13]:

Controller Rule *i*: IF $x_1(t)$ is M_{i1} and $\cdots$ and $x_g(t)$ is M_{ig}

$$\text{THEN } \boldsymbol{U}(t) = -\boldsymbol{K}_i \boldsymbol{X}(t),\quad i = 1 \ldots r, \tag{3.24}$$

where $\boldsymbol{K}_i$ is the local feedback gain matrix and the final model-based controller is

$$\boldsymbol{U}(t) = -\frac{\sum_{i=1}^{r} w_i(t)\boldsymbol{K}_i\boldsymbol{X}(t)}{\sum_{i=1}^{r} w_i(t)} = -\sum_{i=1}^{r} h_i(t)\boldsymbol{K}_i\boldsymbol{X}(t). \tag{3.25}$$

The overall closed-loop system model, which was obtained by combining Eqs. (3.21) and (3.25), is

$$\dot{\boldsymbol{X}}(t) = \sum_{i=1}^{r}\sum_{l=1}^{r} h_i(t)h_l(t)[(\boldsymbol{A}_i - \boldsymbol{B}_i\boldsymbol{K}_l)\boldsymbol{X}(t)] + \boldsymbol{E}_i\boldsymbol{\varphi}(t) . \tag{3.26}$$

A very typical and important stability condition for (3.26) is discussed in the stable of the large if there appears a common and positive definite matrix which is represented with $\boldsymbol{P}$ and feedback gains $\boldsymbol{K}$ such that (3.27) and (3.28) are nearly satisfied:

$$(\boldsymbol{A}_i - \boldsymbol{B}_i\boldsymbol{K}_i)^{\mathrm{T}}\boldsymbol{P} + \boldsymbol{P}(\boldsymbol{A}_i - \boldsymbol{B}_i\boldsymbol{K}_i) + \frac{1}{\eta^2}\boldsymbol{P}\boldsymbol{E}_i\boldsymbol{E}_i^{\mathrm{T}}\boldsymbol{P} < 0 , \tag{3.27}$$

$$\left[\frac{(\boldsymbol{A}_i - \boldsymbol{B}_i\boldsymbol{K}_l) + (\boldsymbol{A}_l - \boldsymbol{B}_l\boldsymbol{K}_i)}{2}\right]^{\mathrm{T}}\boldsymbol{P} + \boldsymbol{P}\left[\frac{(\boldsymbol{A}_i - \boldsymbol{B}_i\boldsymbol{K}_l) + (\boldsymbol{A}_l - \boldsymbol{B}_l\boldsymbol{K}_i)}{2}\right] + \frac{1}{\eta^2}\boldsymbol{P}\boldsymbol{E}_i\boldsymbol{E}_i^{\mathrm{T}}\boldsymbol{P} < 0, \tag{3.28}$$

with the condition $\boldsymbol{P} = \boldsymbol{P}^{\mathrm{T}} > 0$, for those $i < l \leq r$ and $i = 1, 2, \cdots, r$.

Proof: By employing the next Lyapunov function candidate for the fuzzy system in

(3.26)

$$\boldsymbol{V}=\boldsymbol{X}^{\mathrm{T}}(t)\boldsymbol{P}\boldsymbol{X}(t), \tag{3.29}$$

the time derivative of $\boldsymbol{V}$ is

$$\dot{\boldsymbol{X}}^{\mathrm{T}}(t)\boldsymbol{P}\boldsymbol{X}(t)+\boldsymbol{X}^{\mathrm{T}}(t)\boldsymbol{P}\dot{\boldsymbol{X}}(t)=\left\{\sum_{i=1}^{r}\sum_{l=1}^{r}h_i(t)\,h_l(t)[(\boldsymbol{A}_i-\boldsymbol{B}_i\boldsymbol{K}_l)\boldsymbol{X}(t)]\;+\boldsymbol{E}_i\boldsymbol{\varphi}(t)\right\}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{X}(t)$$

$$+\,\boldsymbol{X}^{\mathrm{T}}(t)\boldsymbol{P}\left\{\sum_{i=1}^{r}\sum_{l=1}^{r}h_i(t)\,h_l(t)[(\boldsymbol{A}_i-\boldsymbol{B}_i\boldsymbol{K}_l)\boldsymbol{X}(t)]+\boldsymbol{E}_i\boldsymbol{\varphi}(t)\right\} \tag{3.30}$$

$$=\sum_{i=1}^{r}\sum_{l=1}^{r}h_i(t)\,h_l(t)\boldsymbol{X}^{\mathrm{T}}(t)[(\boldsymbol{A}_i-\boldsymbol{B}_i\boldsymbol{K}_l)^{\mathrm{T}}\boldsymbol{P}+\boldsymbol{P}(\boldsymbol{A}_i-\boldsymbol{B}_i\boldsymbol{K}_l)]\boldsymbol{X}(t)$$

$$+\boldsymbol{\varphi}^{\mathrm{T}}(t)\boldsymbol{E}_i^{\mathrm{T}}\boldsymbol{P}\boldsymbol{X}(t)+\boldsymbol{X}^{\mathrm{T}}(t)\boldsymbol{P}\boldsymbol{E}_i\boldsymbol{\varphi}(t)\;-[\eta^2\boldsymbol{\varphi}^{\mathrm{T}}(t)\boldsymbol{\varphi}(t)+\frac{1}{\eta^2}\boldsymbol{X}^{\mathrm{T}}(t)\boldsymbol{P}\boldsymbol{E}_i\boldsymbol{E}_i^{\mathrm{T}}\boldsymbol{P}\boldsymbol{X}(t)\;]^{*}$$

$$+\left[\eta^2\boldsymbol{\varphi}^{\mathrm{T}}(t)\boldsymbol{\varphi}(t)+\frac{1}{\eta^2}\boldsymbol{X}^{\mathrm{T}}(t)\boldsymbol{P}\boldsymbol{E}_i\boldsymbol{E}_i^{\mathrm{T}}\boldsymbol{P}\boldsymbol{X}(t)\right] \tag{3.31}$$

$$\le\sum_{i=l=1}^{r}h_i^2(t)\,\boldsymbol{X}^{\mathrm{T}}(t)[(\boldsymbol{A}_i-\boldsymbol{B}_i\boldsymbol{K}_i)^{\mathrm{T}}\boldsymbol{P}+\boldsymbol{P}(\boldsymbol{A}_i-\boldsymbol{B}_i\boldsymbol{K}_i)+\frac{1}{\eta^2}\boldsymbol{P}\boldsymbol{E}_i\boldsymbol{E}_i^{\mathrm{T}}\boldsymbol{P}]\boldsymbol{X}(t)$$

$$+\,2\sum_{i<l}^{r}\;h_i(t)h_l(t)\boldsymbol{X}^{\mathrm{T}}(t)\left\{\left[\frac{(\boldsymbol{A}_i-\boldsymbol{B}_i\boldsymbol{K}_l)+(\boldsymbol{A}_l-\boldsymbol{B}_l\boldsymbol{K}_i)}{2}\right]^{\mathrm{T}}\boldsymbol{P}\right.$$

$$\left.+\boldsymbol{P}\left[\frac{(\boldsymbol{A}_i-\boldsymbol{B}_i\boldsymbol{K}_l)+(\boldsymbol{A}_l-\boldsymbol{B}_l\boldsymbol{K}_i)}{2}\right]+\frac{1}{\eta^2}\boldsymbol{P}\boldsymbol{E}_i\boldsymbol{E}_i^{\mathrm{T}}\boldsymbol{P}\right\}\boldsymbol{X}(t)\;+\eta^2\left\|\boldsymbol{\varphi}_{\mathrm{up}}(t)\right\|^2, \tag{3.32}$$

where "$*$" is represented as $-\left[\frac{1}{\eta}(\boldsymbol{P}\boldsymbol{E}_i)^{\mathrm{T}}\boldsymbol{X}(t)-\eta\boldsymbol{\varphi}(t)\right]^{\mathrm{T}}\left[\frac{1}{\eta}(\boldsymbol{P}\boldsymbol{E}_i)^{\mathrm{T}}\boldsymbol{X}(t)-\eta\boldsymbol{\varphi}(t)\right]<0$.

The Evolved Bat Algorithm (EBA) is already discussed and proposed by Tsai et al. in 2012 for the bat complex system in nature. Unlike other swarm intelligence algorithms, a strong group of EBAs is that it has only one variable needed to decide before solving algorithmic problem. Choosing a different environment determines the different size of the search stage in the development process. In this current study, we choose air as an environment because it is an original means of existence in the natural environment where bats live. The operation principle of EBA can be summarized in four steps:

Initialization: the synthetics are distributed in the solution space by means of assigning them random coordinates.

Motion: synthetics are transported according to (3.33) and (3.34) given as follows. A random number should be generated and then checked to see if it is greater than the constant pulse release rate. If the result is positive, the artificial environment is moved through a random procedure as defined in (3.35):

$$x_i^t=x_i^{t-1}+D, \tag{3.33}$$

where x_i^t means the coordinate of the kind of i^{th} artificial agent at t^{th} iteration, in which x_i^{t-1} denote the coordinates in the last iteration, and the parameter D is the distance traveled by the artificial environment in this iteration,

$$D=\gamma\cdot\Delta T, \tag{3.34}$$

where the parameter γ is regarded as a constant which is corresponding to the environment in which they are chosen in the numerical experiments, $\gamma = 0.17$ is employed, and parameter $\Delta T \in [-1, 1]$ is a random number,

$$x_i^{t_R} = \beta\left(x_{\text{best}} - x_i^t\right), \quad \beta \in [0, 1], \tag{3.35}$$

where β is a random number, x_{best} is the coordinate of the best agent, and $x_i^{t_R}$ is the new coordinate of the artificial agent in the framework of the random walk process. The detailed algorithm is referenced in [14].

4. Example

In order to show how the theory from the previous sections is applied in the controller design, we consider the discrete time-delay fuzzy system

$$X(k+1) = \left(h_1\begin{bmatrix} 0.6 & 0.4 \\ -0.8 & 0.2 \end{bmatrix} + h_2\begin{bmatrix} 0.1 & -0.1 \\ 0.2 & 0.2 \end{bmatrix}\right)X(k) + \left(h_1\begin{bmatrix} 0.08 & -0.8 \\ -0.01 & -0.05 \end{bmatrix} + h_2\begin{bmatrix} 0.01 & 0.2 \\ 0.01 & -0.05 \end{bmatrix}\right)X(k-1)$$

$$+\left(h_1\begin{bmatrix} -0.205 & 0.4 \\ 0.05 & 0.405 \end{bmatrix} + h_2\begin{bmatrix} 0.05 & -0.1 \\ 0 & -0.1 \end{bmatrix}\right)X(k-2)$$

$$+ \Delta A X(k) + \Delta A_{d1} X(k-1) + \Delta A_{d2} X(k-2), \tag{4.1a}$$

where

$$\|\Delta A\| \le 0.4749, \ \|\Delta A_{\text{d1}}\| \le 0.0186, \ \|\Delta A_{\text{d2}}\| \le 0.016. \tag{4.1b}$$

The purpose of this is to check if Eq. (4.1) meets pole locations inside the specified disk $D(0.2,\ 0.7)$ [8].

In order to analyze the accuracy and calculation speed of the theoretical results presented in Section 3, three of these test functions are listed relation with (4.1) to be used in the experiments. The results are compared with the original EBA. The optimization goal related to the test functions is to minimize the objective functions.

According to (3.2) and (4.1b), the uncertainties are given $\beta = 0.4749,\ \eta_1 = 0.0186$ and $\eta_2 = 0.016$.

The values $h_1(k) = 0.2$ and $h_2(k) = 0.8$ are chosen. Moreover, according to (3.7), the result is

$$d_1 = \frac{1}{r}\left(\beta + \sum_{i=1}^{2}\left(\|A_{di}\| + \eta_i\right)\left(r - |\alpha|\right)^{-h_i}\right) = 1.2264 > d_s = \rho\left(\frac{A - \alpha I}{r}\right) = 1. \tag{4.2}$$

Therefore, (3.7) is not satisfied.

The simulation results for $h(g)$ in (3.18), where $g = e^{j\theta}$ for $\theta \in [0, 2\pi]$, are given in Fig. 3 revealing that $h(g) < d_s = 1$. We can conclude Eq. (4.1) is $D(0.2,\ 0.7)$-stable by Corollary 3.1.

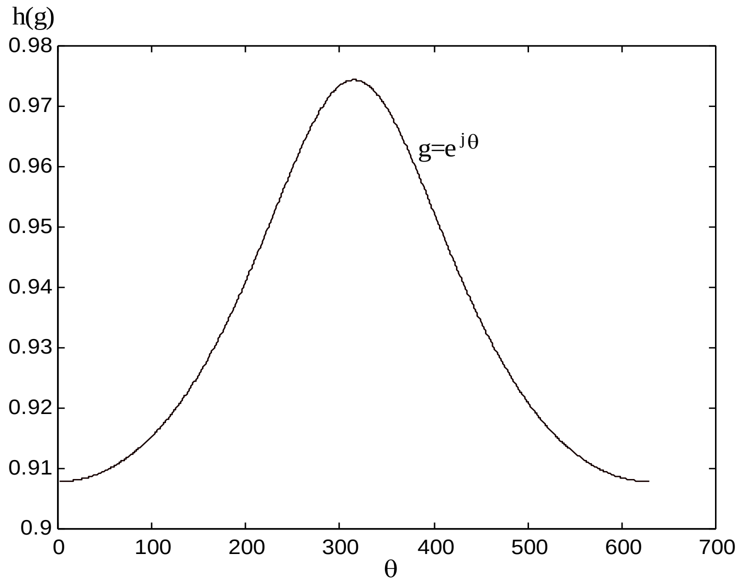

Fig. 3. Simulation results for *h*(*g*)

To verify this conclusion, a set of uncertainty factors that satisfy the standard conditions have been identified in relation with (4.1b)

$$\Delta A_1 = \begin{bmatrix} 0.305 & 0.15 \\ 0 & 0.2 \end{bmatrix},\ \Delta A_2 = \begin{bmatrix} 0.1 & -0.2 \\ 0.2 & 0.5 \end{bmatrix},\ \Delta A_{d11} = \begin{bmatrix} -0.02 & 0.14 \\ 0.04 & 0.05 \end{bmatrix}$$

$$\Delta A_{d12} = \begin{bmatrix} 0.02 & -0.02 \\ -0.01 & 0 \end{bmatrix},\ \Delta A_{d21} = \begin{bmatrix} 0.455 & -0.8 \\ 0.755 & 0.01 \end{bmatrix},\ \Delta A_{d22} = \begin{bmatrix} -0.1 & 0.2 \\ 0.2 & 0.01 \end{bmatrix}. \quad (4.3)$$

Fig. 4 shows the poles $(0.5299 \pm j0.1218,\ -0.0585 \pm j\,0.164,\ 0.019 \pm j\,0.1091)$ lie inside the specific disk in the parameter $D(0.2,\,0.7)$. So, these so-called fuzzy complex systems in relation with (4.1) must meet the time-domain of specifications in the presence of parametric uncertainties given in (4.3). This result justifies our current results.

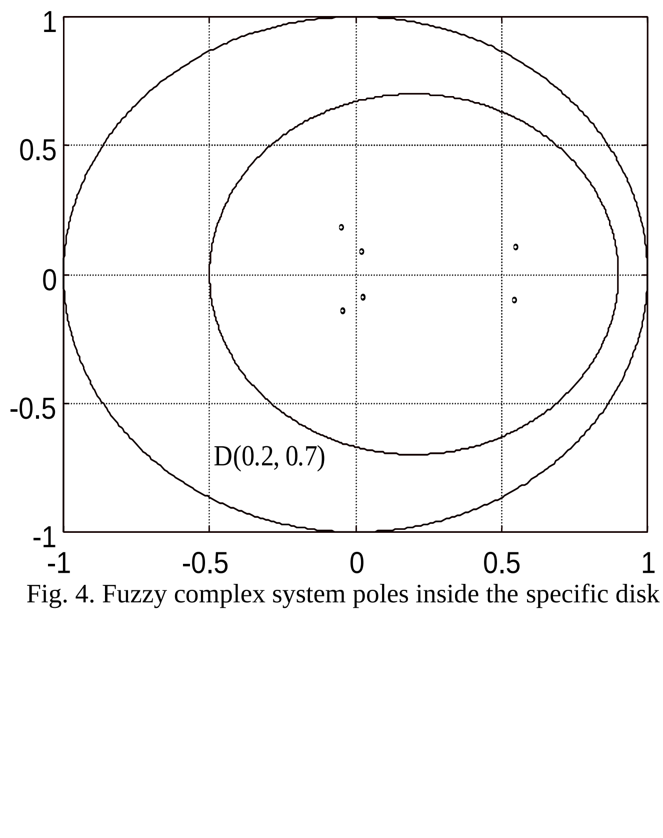

Fig. 4. Fuzzy complex system poles inside the specific disk

In addition, EBA is next applied *to find the right solutions. In this case, search engine solutions can be characterized as feasible and unenforceable solutions. This means that the fitness function must be better designed in a two-step process to meet the needs of this application. We thus design the fitness functions according to the stability criterion derived from the LMI conditions using the Lyapunov approach. AND as a a logical function is used to generate a double sorting result in the solutions obtained. Fitness activity is listed as

$$F\left(\boldsymbol{P},\ \boldsymbol{K}\right)=\begin{cases}1, & \text{if } \left(\boldsymbol{A}_i-\boldsymbol{B}_i\boldsymbol{K}\right)^{\mathrm{T}}\boldsymbol{P}+\boldsymbol{P}\left(\boldsymbol{A}_i-\boldsymbol{B}_i\boldsymbol{K}\right)>0 \text{ and } \boldsymbol{P}=\boldsymbol{P}^{\mathrm{T}}>0\\ 0, & \text{otherwise}\end{cases} \quad . \tag{4.4}$$

The current common and positive definite matrix, i.e., the parameter $\boldsymbol{P}$, is always limited to being symmetric when the EBA is used to modify its elements. In addition, the initial conditions are used in the initial process for both states $\boldsymbol{P}$ and $\boldsymbol{K}$. Matrix $\boldsymbol{P}$ is thus kept in influencing via the same range in the boundary conditions for producing and manufacturing the feasible solutions in a very and suitable range, but not been applied for the matrix $\boldsymbol{K}$ owing that the total effect have been contributed to the fuzzy complex system by means of the those control forces is relatively and very small. The parameters employed in the numerical experiment for EBA are listed in Table 1.

Table 1: Parameters for EBA

Boundary condition for matrix $\boldsymbol{P}$ and $\boldsymbol{K}$	[−5, 5]
Environment material	Air
Number of iterations	500
Population size	16
Number of runs	25

Usually, the algorithm needs many iterations to find the next best solution. Therefore, the same numerical experiment should be regarded as a repeated several times to be sure that converged results are consistent. The execution times listed in Table 1 are used to present a series of the numerical and experimental results to test of the kinds of results obtained are statistically consistent. Informational materials like air are chosen because they are suitable for five room club in the natural environment. The total volume represents the number of compounds used simultaneously in the solution space for each iteration. In the experiment, we set the population size to 16. The possible solutions of the s number that EBA had a different gel are shown in Fig. 5. Table 2 shows a statistical analysis of the results obtained from EBA for more than 25 runs. The results obtained in a given experiment found a number of possible solutions that is far more than sufficient to determine the application in its complex from more than the number of system variables. The results are shown in Table 2 and Fig. 5 The applicable solutions are more flexible and stable [11], [13], [14]. All data analyzed in this study have been included in this paper.

5. Conclusion

The results of the analysis of the example indicate the benefit of dealing with the complex plants and questionable conditions using a smart controller (specifically an FLC). In addition to the formal solution of the FLC and the model-based arrangement that can be easily found in the criterion of controller design, its increasingly impressive elements are translations of the semantic fuzzy expressions and the estimation of nonlinear programming.

The fuzzy controller can be adjusted through pragmatic perception or empirical methods, thereby eliminating the multi-faceted nature of the plants. In this analyzed concept, a complex non-linear plant is shown and further changes have been made to show the advantages of the basic heuristically adjusted FLC in terms of stability and strength compared with the non-optimal fuzzy and non-fuzzy control design.

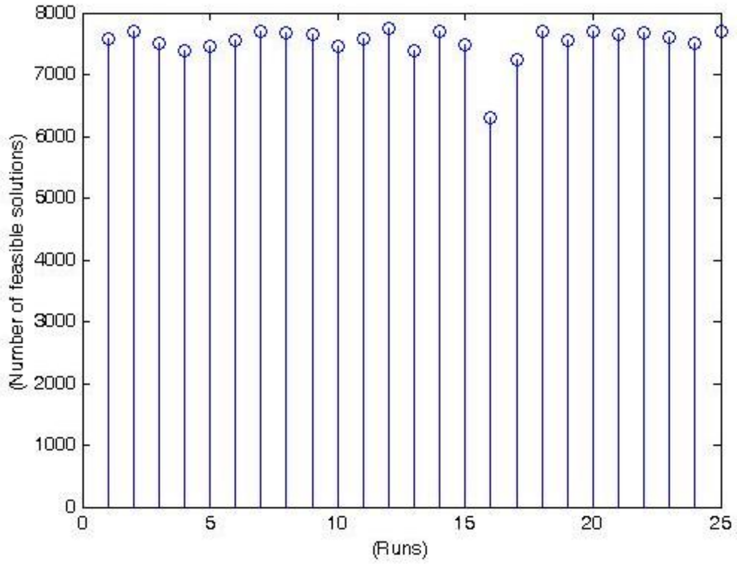

Fig. 5. Feasible solutions found by EBA in 25 runs

Table 2: Statistical analysis of the obtained feasible solutions

Mean	Minimum	Maximum	Mode
7528	6291	7748	7576

A novel and important approach to the control of nonlinear systems which can be applied in chaotic and limited cycles was regarded and realized in a fuzzy controller and an appropriate controller design in this paper. If the fuzzy controller cannot stabilize the system, an optimization of fuzzy control is utilized to adjust the models for the nonlinear system and then the system is stabilized asymptotically by regulating the control parameters.

In this paper, a fuzzy controller was proposed via the Parallel Distributed Compensation technique in order to stabilize and check the nonlinear systems stability. The application is also practical for the control of chaotic systems and limit cycles.

References

[1] K. TANAKA, and M. SUGENO, *Stability analysis and design of fuzzy control systems*, Fuzzy Sets and Systems, **45**, pp. 135–156, 1992.

[2] K. TANAKA, and M. SANO, *A robust stabilization problem of fuzzy control systems and its application to backing up control of a truck-trailer,* IEEE Transactions on Fuzzy Systems, **2**, pp. 119–134, 1994.

[3] J. ZHAO, V. WERTZ, and R.GOREZ, *Linear T-S fuzzy model based robust stabilizing controller design*, Proc. 34th IEEE Conference on Decision and Control, New Orleans, LA, USA, pp. 255–260, 1995.

[4] H. O. WANG, K.TANAKA, and M. F. GRIFFIN, *An approach to fuzzy control of nonlinear systems: stability and design issues*, IEEE Transactions on Fuzzy Systems, **4**, pp. 14–23, 1996.

[5] D. HINRICHSEN AND A. J. PRITCHARD, *New robustness results for linear system under real perturbations*, Proc. 27th IEEE Conference on Decision and Control, Austin, TX, USA, pp. 1375–1379, 1988.

[6] V. L. KHARITONOV, *Stability radii and global stability of difference systems*, Proc. 30th IEEE Conference on Decision and Control, Brighton, UK, pp. 877–880, 1991.

[7] M. VIDYASAGAR, *Control System Synthesis: A Factorization Approach*, MIT Press, Cambridge, MA, USA, 1985.

[8] S. H. LEE and T. T. LEE, *Optimal pole assignment for a discrete linear regulator with constant disturbances*, International Journal of Control, **45**, pp. 161–168, 1987.

[9] C. W. CHEN, *A criterion of robustness intelligent nonlinear control for multiple time-delay systems*

based on fuzzy Lyapunov methods, Nonlinear Dynamics, **76**(1), pp. 13–22, 2014.

[10] C. W. CHEN, *A criterion of robustness intelligent nonlinear control for multiple time-delay systems based on fuzzy Lyapunov methods*, Nonlinear Dynamics, **76**(1), pp. 23–31, 2014.

[11] T. CHEN, S. KHURRAM, and C. CHENG, *A relaxed structural mechanics and fuzzy control for fluid–structure dynamic analysis*, Eng. Comput. **36**(7), pp. 2200–2219, 2019.

[12] T. CHEN, *Evolved fuzzy NN control for discrete-time nonlinear systems*, J. Circuits Syst. Comput., **29**(1), 2050015.

[13] T. CHEN, *On the algorithmic stability of optimal control with derivative operators*, Circuits Syst. Signal Process., doi:10.1007/s00034-020- 01447-1, 2020.

[14] P.W. TSAI, T. HATAT, B. AHMAD, and C.W.CHEN, *Structural system simulation and control via NN based fuzzy model*, Struct. Eng. Mech., **56**(3), pp. 385–407, 2015.

[15] T. CHEN, *An intelligent algorithm optimum for building design of fuzzy structures*, Iran J. Sci. Technol, Trans Civ. Eng., **44**, pp. 523–531, 2019.

[16] T. HAIDEGGER, L. KOVÁCS, R.-E. PRECUP, S. PREITL, B. BENYÓ, and Z. BENYÓ, *Cascade control for telerobotic systems serving space medicine*, IFAC Proceedings Volumes, **44**(1), pp. 3759–3764, 2011.

[17] A. R. CHATTERJEE, F. CHATTERJEE, and T. ENDO, *Augmented stable fuzzy control for flexible robotic arm using LMI approach and neuro-fuzzy state space modeling*, IEEE Transactions on Industrial Electronics, **55**, pp. 1256–1270, 2008.

[18] R.-E. PRECUP and M. L. TOMESCU, *Stable fuzzy logic control of a general class of chaotic systems*, Neural Computing and Applications, **26**(3), pp. 541–550, 2015.

[19] R. P. ALVAREZ GIL, Z. C. JOHANYAK, and T. KOVACS, *Surrogate model based optimization of traffic lights cycles and green period ratios using microscopic simulation and fuzzy rule interpolation,* International Journal of Artificial Intelligence, **16**(1), pp. 20–40, 2018.

[20] P. W. TSAI, A. ALSAEDI, T. HATAT, and C. W. CHEN, *A novel control algorithm for interaction between surface waves and a permeable floating structure*, China Ocean Eng., **30**, pp. 161–176, 2016.