Evolving Fuzzy Models of Shape Memory Alloy Wire Actuators

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Abstract. This paper suggests Takagi-Sugeno-Kang (TSK) fuzzy models that characterize the position of Shape Memory Alloy (SMA) wire actuators. The systems dynamics are important because, since SMA wire actuators are included in control systems in various critical applications, the position control systems have to ensure very good performance indices. The output of these systems, viewed as controlled processes, is the position of the SMA wire actuators, and the input is the current signal which supplies the actuator, i.e., it flows through the wire. Starting with these two input and output signals directly related to the process, different numbers of additional TSK fuzzy model inputs and outputs obtained from past inputs and/or outputs are considered. The structures and parameters of the fuzzy models are evolved by an incremental online identification algorithm. Two evolving TSK fuzzy models are derived, they are next tested against the experimental data and compared. The experimental results indicate that the proposed fuzzy models are consistent with training and validation data.

1. Introduction

As analyzed in [1], the analysis of the state-of-the-art of control theory and applications related to Shape Memory Alloy (SMA) actuators shows that they are in continuous progress because of their special features. A comprehensive review on the challenges for practical applications of SMA actuators is given in [2]. As all actuators, the SMA ones have to ensure very good control system performance indices as they are integrated in control systems in various critical applications. The special features of SMA actuators, which justify their success, are maintaining a deformed shape until heat is applied to recover the original shape [3], capability to high strain recovery and withstanding the higher load and high damping and supporting large reversible changes of mechanical and physical characteristics [4], [5].

Due to the above features and the need to validate the control system structure, testing equipment was designed in both simple cases to demonstrate the SMA behavior and complicated ones
to model the specific processes and next to design controllers including nonlinear robust ones for position and tracking control. Suggestive examples of testing equipment are reported in [6]–[8].

SMA actuators are implemented and validated by means of both simulation and experimental results in a wide variety of applications as biomedical engineering [9], industry, vibration control systems [4], [5], [9], [10]–[13], aeronautics and aviation [6], [14]. The recent literature highlights their attractiveness in robotics control system especially for hand prostheses which can follow a two-dimensional motion with a relatively high accuracy driven by SMA coils [15] and for the development of a low-cost five-fingered prosthetic hand because they ensure a biological-like behavior that reacts fast but smoothly as the natural muscular fibers [16]. A bidirectional SMA rotating actuator using a rotating frame and two SMA wire-based actuating units similar to human skeletal muscle systems was designed in [17] to provide angular displacements in both clockwise and counter-clockwise directions with compliance.

The modeling of SMA actuators is a hot topic in the framework of model-based design. In addition, since the experiments are expected to have certain costs, accurate process models are important in order to ensure the convenient digital simulation of SMA actuators as controlled process and the control systems as well. The mathematical model of SMA as a dynamical system is highly nonlinear, which makes it complicated to design an appropriate controller. SMAs can exist in two different phases depending on the applied temperature and stress. The difference between the heating and the cooling transition gives rise to hysteresis. Therefore, for modeling the SMA behavior it is more convenient to use several system identification approaches implemented in Matlab’s System Identification Toolbox in order to describe the system model as a linear one with uncertainties and then apply the robust stability method to control the system efficiently as illustrated in [6] and [18]. Other two approaches to SMA model design implementation are based on Liang’s constitutive model and discussed in [10]; the first one is a stress-driven model and the second one is a strain-driven model (used only to implement the SMA-based actuator). In stress-driven model, stress and temperature are inputs, whereas strain is output. In strain-driven model, temperature and strain are inputs, whereas stress is output. In [14], the model of SMA actuator system consists of four dynamic equations including the temperature dynamics based on thermal energy conservation equation, phase transformation equation between martensite and austenite, SMA constitutive law and the mechanical dynamics. The discrete-time nonlinear dynamical system described in the Single-Input-Single Output (SISO) form is equivalently represented in [7] as an equation by linearization using Taylor’s formula around the origin, obtaining a linear dynamic model along with its unmodeled dynamics.

Although all the modeling approaches briefly discussed above ensure good modeling performance, there is still room for performance improvement. One direction in this regard is to develop nonlinear models of SMA actuators. Representative nonlinear models are the fuzzy models as they are relatively easily understandable and also may embed the human expert’s knowledge in dealing with the processes they model aiming their control. As specified in [19], the concept of evolving fuzzy (or rule-based) controllers was proposed by Angelov back in 2001 and further developed in his later works [20]–[25]. These controllers employ evolving Takagi-Sugeno-Kang (TSK) fuzzy models, which are characterized by computing the rule bases by a learning process, i.e., by continuous online rule base learning. Some recent illustrative results on evolving fuzzy models are offered in [26]–[29]. The TSK fuzzy models are developed in terms of evolving the model structures and parameters by means of online identification algorithms.

This paper is built upon authors’ recent papers on evolving TSK fuzzy models [19], [30]–[32] mainly focusing on modeling the midcarpal joint angles in the framework of human hand
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This paper treats the following topics: the main implementation details of the online identification algorithm are described in the next section. The development of evolving TSK fuzzy models and the experimental results are presented in Section 3. The conclusions are drawn in Section 4.

2. Implementation details of online identification algorithm

The online identification algorithm is implemented by adapting the theory treated in [33], with the aid of the eFS Lab software developed by Dourado and his team according to the description given in [34] and [35]. The flowchart of the incremental online identification algorithm is presented in Fig. 1. This algorithm is organized in terms of the following steps, also given in [19] and authors’ recent papers highlighted in the References section:

![Flowchart of incremental online identification algorithm](image-url)

Fig. 1. Flowchart of incremental online identification algorithm [19], [36].
Step 1. The structure of the rule base is initialized by setting all parameters of the rule antecedents in order to first contain one rule, i.e., \( n_R = 1 \), where \( n_R \) is the number of rules. The subtractive clustering is next applied to compute the parameters of the evolving TSK fuzzy models using the first data point \( p_1 \) represented as a vector in \( \mathbb{R}^{n+1} \). The general expression of the data point \( p \) at the discrete time step \( k \) (and also the index of the current sample) is \( p_k \), and this point and also vector belongs to the input-output data set \( \{ p_k | k = 1 \ldots D \} \subset \mathbb{R}^{n+1} \) [19]

\[
p_k = [p_k^1_p^2 \ldots p_k^{n+1}]^T
\]

\[
p = [z^T y]^T = [z_1 \ z_2 \ \ldots \ z_n \ y_1]^T = [p^1 \ p^2 \ \ldots \ p^n \ p^{n+1}]
\]

where \( D \) is the number of input-output data points or data points or data samples or samples, \( z \) is the input vector, and the superscript \( T \) indicates matrix transposition. The rule base of TSK fuzzy models with affine rule consequents is [19]

Rule \( i \) : IF \( z_1 \) is \( LT_{i1} \) AND ... AND \( z_n \) is \( LT_{in} \)

THEN \( y_i^l = a_{i0} + a_{i1}z_1 + \ldots + a_{in}z_n, \ i = 1\ldots n_R, \)

where \( z_j, \ j = 1\ldots n, \) are the input (or scheduling) variables, \( n \) is the number of input variables, \( LT_{ij}, \ i = 1\ldots n_R, \ j = 1\ldots n, \) are the input linguistic terms, \( y_i^l \) is the output of the local model in the rule consequent of the rule \( i, \ i = 1\ldots n_R, \) and \( a_{i\chi}, \ i = 1\ldots n_R, \ \chi = 0\ldots n, \) are the parameters in the rule consequents.

Using the notation \( \pi_i \) for the parameter vector of the rule \( i, \ i = 1\ldots n_R \) [19]

\[
\pi_i = [a_{i0} \ a_{i1} \ a_{i2} \ \ldots \ a_{in}]^T, \ i = 1\ldots n_R,
\]

the algebraic product t-norm as an AND operator in the inference engine, and the weighted average defuzzification method in TSK fuzzy model structure, the expression of TSK fuzzy model output \( y_i \) is [19]

\[
y_i = \sum_{i=1}^{n_R} \sum_{j=1}^{n} \frac{\tau_i y_i^j}{\tau_i} = \sum_{i=1}^{n_R} \frac{\lambda_i y_i^j}{\tau_i} y_i^j = [z^T \pi] \lambda_i = \sum_{i=1}^{n_R} \frac{\tau_i}{\sum_{i=1}^{n_R} \tau_i}, \ i = 1\ldots n_R
\]

where the firing degree of the rule \( i \) and the normalized firing degree of this rule are \( \tau_i(z) \) and \( \lambda_i \), respectively. The firing degree of the rule \( i \) is [19]

\[
\tau_i(z) = AND(\mu_{i1}(z_1), \mu_{i2}(z_2), \ldots, \mu_{in}(z_n)) = \mu_{i1}(z_1) \cdot \mu_{i2}(z_2) \cdot \ldots \cdot \mu_{in}(z_n), \ i = 1\ldots n_R.
\]

Several parameters specific to the incremental online identification algorithm are initialized. The initialization is carried out according to the following recommendations given in [33]:

\[
\hat{\theta}_1 = [\pi_1^T \pi_2^T \ldots \pi_{n_R}^T]^T = [0 \ 0 \ \ldots \ 0]^T, \ C_k = \Omega I, \quad r_k = 0.4, \quad k = 1, \quad n_R = 1, \quad z_k^1 = z_k, \quad P_k^1(p_k^1) = 1,
\]

where \( C_k \in \mathbb{R}^{n_R(n+1) \times n_R(n+1)} \) is the fuzzy covariance matrix related to the clusters, \( I \) is the \( n_R(n+1)^{th} \) order identity matrix, \( \Omega = \text{const} \), \( \Omega > 0 \), is a relatively large number, \( \hat{\theta}_k \) is an
estimation of the parameter vector in the rule consequents at the discrete time moment (and also data sample index) \( k \), and \( r_s, r_s > 0 \), is the spread of the Gaussian input membership functions \( \mu_{ij}, i = 1...n_R, j = 1...n \), of the fuzzy sets of the input linguistic terms LTI \( i, j \) [19]

\[
\mu_{ij}(z_j) = e^{-\frac{4(z_j - z^*_j)^2}{r^2_s}}, \quad i = 1...n_R, \quad j = 1...n
\]  

(7)

\( z^*_j, i = 1...n_R, j = 1...n \), are the membership function centers, \( p^*_1 \) in (7) is the first cluster center, \( z^*_1 \) is the center of the rule 1 and also the projection of \( p^*_1 \) on the axis \( z \) in terms of (2), and \( P_i(p^*_1) \) is the potential of \( p^*_1 \).

Step 2. The data sample index \( k \) is incremented, namely replaced with \( k + 1 \), and the next data sample \( p_k \) that belongs to the input-output data set \( \{ p_k | k = 1...D \} \subset \mathbb{R}^{n+1} \) is read.

Step 3. The potential of each new data sample \( P_k(p_k) \) and the potentials of the centers \( P_k(p^*_\eta) \) of existing rules (clusters) with the index \( \eta \) are recursively updated according to the formulae given in [19] and [33].

Step 4. The possible modification or upgrade of the rule base structure is done using the potential of the new data (point or pair) in comparison with that of the existing rules’ centers. The rule base structure is modified if certain conditions specified in [33] are fulfilled.

Step 5. The parameters in the rule consequents are updated using either the Recursive Least Squares (RLS) algorithm or the weighted Recursive Least Squares (wRLS) algorithm. These algorithms, which can also be viewed as learning algorithms, lead to the updated vectors \( \hat{\theta}_k \) (i.e., an estimation of the parameters in the rule consequents at the discrete time step \( k \)) and \( C_k, k = 2...D \). Several details on RLS are given in [19] and [33].

Step 6. The output of the evolving Takagi-Sugeno-Kang fuzzy model at the next discrete time step \( k+1 \) is predicted and expressed as \( \hat{y}_{k+1} \) [19]

\[
\hat{y}_{k+1} = \psi^T_k \hat{\theta}_k,
\]

(8)

with the general notations (applied to any element of the data set)

\[
\begin{align*}
y &= \psi^T \theta, \quad \theta = [ \pi_1^T \pi_2^T \ldots \pi_{n_R}^T ]^T, \\
\psi^T &= [ \lambda_1 [1 z^T] \lambda_2 [1 z^T] \ldots \lambda_{n_R} [1 z^T] ].
\end{align*}
\]

(9)

Step 7. The algorithm continues with step 2 until all data points of the input-output data set \( \{ p_k | k = 1...D \} \) are read.

3. Evolving Takagi-Sugeno-Kang fuzzy models

The output of all evolving TSK fuzzy models is \( y_k \), which represents the position of the SMA wire actuators. In this regard, the generic expression of a TSK fuzzy model is

\[
y_k = f(z_k),
\]

(10)

where \( f \) is the nonlinear input-output map of the TSK fuzzy model. As specified in sections 2 and 3, the input vector \( z_k \) of the TSK fuzzy models contains as an essential element the current signal which supplies the actuator (it flows through the wire). Additional TSK fuzzy model inputs and outputs obtained from past inputs and/or outputs are included in \( z_k \), thus modifying the model structures and leading to different TSK fuzzy models. In this regard, the TSK fuzzy
models generally expressed in (10) can be considered as Nonlinear AutoRegressive eXogenous (NARX) models.

The first system input, namely the current signal which supplies the actuator, with the notation \( u_k \), was generated by processing the input-output data given in [6] using a sampling period of 0.01 s. The number of input-output data pairs used in training is \( D = 500 \), which covers a time horizon of 5 s. The number of input-output data pairs used in validation is \( D = 2701 \), which covers a time horizon of 27.01 s. It is supposed that these ranges of magnitudes and frequencies used in [6] actually cover and capture various process (i.e., SMA wire actuator) dynamics, which are next important in the operation of the position control system. The evolution of the system input versus time is presented in Figs. 2 and 3, which include the input data for both training and validation.

![Fig. 2. System input \( u_k \) (mA) versus time for training data.](image1)

![Fig. 3. System input \( u_k \) (mA) versus time for validation data.](image2)
The evolution of the system output will be presented in the next figures along with the evolutions of the model outputs in order to ensure a convenient comparison of the outputs and visually assess the performance/quality of the two nonlinear models derived in this paper. The graphs presented in Figs. 2 and 3 illustrate the nature of the input values. They illustrate the noisy nature of the signals, which require and justify nonlinear models, and also the fact that the simplification of the models can be achieved. The real system output values will be presented as follows along with the model output.

The past input and output values were actually obtained by shifting the training and validation data samples. Several fuzzy models were tested in this regard but the two ones are described as follows because they are the simplest ones as far as their numbers of parameters are concerned. The first TSK fuzzy model operates with the input vector

$$z_k = \begin{bmatrix} u_k & u_{k-1} & u_{k-2} & y_{k-1} \end{bmatrix}^T,$$

and the second TSK fuzzy model operates with the input vector

$$z_k = \begin{bmatrix} u_k & u_{k-1} & y_{k-1} & y_{k-2} \end{bmatrix}^T.$$

The application of the online identification algorithm presented in the previous section with the RLS algorithm involved in step 5 gave the following results: the first TSK fuzzy model evolved to \( n_R = 9 \) rules and the number of evolved parameters is 117; the second TSK fuzzy model evolved to \( n_R = 1 \) rule and the number of evolved parameters is 13.

As expected and shown in [19] for a different nonlinear system, the TSK fuzzy models and their performance depend on the number of input variables. Different model structures, i.e., different numbers of rules resulting in different number of evolved (or identified parameters) are obtained for different input variables.

The real-time experimental results on the training data set are presented in Fig. 4 for the first TSK fuzzy model and Fig. 5 for the second TSK fuzzy model. The real-time experimental results on the validation data set are presented in Fig. 6 for the first TSK fuzzy model and Fig. 7 for the second TSK fuzzy model. The results in Figs. 4 to 7 are given as responses of \( y \) versus time of the TSK fuzzy models and the real-world system, i.e., the SMA wire actuator. The additional subscript \( d \) is inserted to the output value in order to highlight the desired value, i.e., the real-world system output \( y_d \) and to differentiate it from the model output \( y \).

Figs. 4 to 7 show that the validation performance is consistent with the training one. In addition, since no numerical performance comparison is carried out, the visual comparison shows that the first model exhibits overall better performance compared to the second one. That is expected as the number of parameters of the first model is higher compared to the second one. The performance can be further improved if the number of inputs is increased. However, as outlined in [19], a trade-off to performance and model complexity should be targeted. The systematic performance assessment and comparison can be done in terms of using adequate performance indices as, for example, those applied in authors’ recent paper [37].

This paper gives results on training and validation data. As highlighted in [19], this could be a methodological issue because the performance of the models is checked and illustrated on validation data. But these models were selected as the best ones for modeling this validation data considering an as reduced as possible number of inputs resulting in a reasonable number of parameters to be trained (or identified). The model performance should be ideally checked in new unseen test data.
The experiments conducted on this challenging process show that the RLS algorithm involved in step 5 of the online identification algorithm ensures better performance compared to the wRLS algorithm. However, this is not valid at all system inputs. Moreover, as pointed out in authors’ recent papers on evolving fuzzy modeling and tensor product-based modeling, the conclusions drawn in this paper cannot be generalized to other processes. Such representative processes, also subjected to model-based and data-driven model-free control, include mobile robots [38], [39], hybrid electric vehicles [40], medical applications [19], [41]–[43], fuzzy models [44]–[46], hardware applications [47], networks [48], data-driven control [49], [50], and manufacturing systems [51]–[53].

![Fig. 4. SMA position versus time of first TSK fuzzy model (dashed line), \( y_k \) (mm) and real-world system (solid line) \( y_{k,d} \) (mm), on training data set.](image1)

![Fig. 5. SMA position versus time of second TSK fuzzy model (dashed line), \( y_k \) (mm) and real-world system (solid line) \( y_{k,d} \) (mm), on training data set.](image2)
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4. Conclusions

This paper suggested two TSK fuzzy models to describe the dynamics specific to the position of SMA wire actuators as a representative and challenging SISO nonlinear system considered as a controlled process. The structures and the parameters of the TSK fuzzy models were evolved by an incremental online identification algorithm, which actually carries out learning nonlinear dynamic models.
The experimental results illustrate that past outputs and inputs improve the model performance. Adding mode inputs would definitely further improve the model performance, but attention should be paid to the trade-off to performance and model complexity because model-based fuzzy control is one of the future research directions.

These TSK fuzzy models are important because they were built using the data reported in [6], which was collected manually. Therefore, the two fuzzy models suggested here are useful nonlinear dynamic models in the digital simulation of the behaviors of SMA wire actuators and allow the convenient testing of various position control system structures.

Future research will be focused on the controller design and tuning for this challenging nonlinear process. The models proposed in this paper will enable the validation of the position control systems by digital simulation of their behaviors in important operating regimes.

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