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Manifold Optimization with MMSE Hybrid Precoder for Mm-Wave Massive MIMO Communication

Divya SINGH[1] and Aasheesh SHUKLA[*]

[1]Department of ECE, GLA University Mathura, UP, India
E-mail: divya.singh@gla.ac.in, Aasheesh.shukla@gla.ac.in[*]
[*] Corresponding author

Abstract. *Hybrid Precoding* (HP) major key aspects of millimeter-wave in wireless communication is used to enhance the energy and spectrum efficiency and also to reduce the system complexity, cost, and path losses. The sub-connected structure of HP is used further to minimize hardware complexity and power consumption. In this article, an effective HP algorithm is developed for the maximization of spectral efficiency as well as for the minimization of bit error rate in the Mm-Wave massive MIMO system. This proposed algorithm makes use of Manifold optimization and MMSE criterion to increase spectral and energy efficiency and also to reduce hardware complexity. Simulation results show the achievement of significant efficiency and cost via the use of the proposed algorithm over conventional ones.

Key-words: Mm-Wave; Manifold optimization; MIMO; Hybrid precoding; Minimum mean square error.

1. Introduction

To meet the demand of the increasing world population for the fastest data transmission over a larger bandwidth, people have carried out their research in the field of millimeter-wave for massive MIMO system. The recent wireless system used for 5G and beyond integrates several technologies in which massive MIMO and Mm-Wave play a vital role [1–3]. These integrated technologies provide lower latency, wider network coverage, higher spectral and energy efficiency, and a larger frequency spectrum. In mm-wave technology, hybrid precoding *i.e.* the arrangement of analog as well as digital precoding is considered as a spectrum efficient and cost-efficient solution [4–6].

Hybrid analog and baseband precoding can solve the low-scattering problem instigated by Mm-Wave signal attenuation. It can also lower the cost as well as power consumption of data converters and RF chains in MIMO transceivers. However, the complexity of baseband Precoder is still quite high for frequency selective based multi-carrier systems [7–9]. The digital Precoder was designed in such a way that complexity can be reduced [10]. The optimization problem of spectrum efficiency is decomposed into several sub-optimization problems to increase efficiency and to reduce complexity in various algorithms [11–14]. An algorithm based on the integration of Particle Filter and Particle swarm optimization to reduce the energy consumption and optimal tuning of fuzzy controllers [15–17]. A Deep Q-learning algorithm along with the Gravitational Search procedure is used to attain better stability in Neural networks, and a fixed interval of time is allotted to each user for tuning to solve the problem of vehicle routing [18, 19]. The simulation results of spectrum efficiency showed improved spectral efficiency for Mm-Wave MIMO with minor performance degradation as compared to the mentioned work in the previous research.

The main contributions of this paper are:

1. An efficient algorithm for hybrid Precoder design is used to enhance the spectral efficiency in the 5G era.
2. The manifold optimization and MMSE are used to resolve the nonconvex problem by considering the perfect CSI and transmission power.
3. Numerical results illustrate that the proposed design of the hybrid Precoder provides good performance for the mm-wave massive MIMO network.

Throughout this paper, light letters like ‘x’, bold letters like ‘X’, and bold letters like ‘x’ are used to denote scalar quantities, matrices, and column vectors respectively. Let $(X)^T$, $(X)^*$, and $(X)^H$ symbolize transpose, conjugate, and matrix (vector) ‘X’ respectively. [X]ij indicates the i,j element of a matrix X. $(X)^{-1}$, $\| X \|$, and $\| X \|_F$ represent inverse, modulus, and Frobenius norm in a respective manner.

The rest of this paper is laid out in the following mode. Section 2 discusses the previous related effort to this paper. In Section 3, the fully-connected mm-wave MIMO system model is introduced and outline the HP optimization problem. Section 4 includes the discussion of the proposed hybrid Precoder design for spectral efficiency maximization. The MMSE problem is solved using this algorithm. Finally, in Section 5, various simulation results are elaborated. Section 6 covers the overall conclusion of this paper.

2. Related Work

A lot of investigation on the HP design has been done for narrowband systems [8, 12]. HP design for the perfect and imperfect channel was studied in [20–22]. In a millimeter-wave network, the paper [23] proposes methodologies for the creation of linear hybrid Precoders for wireless sensors with de-centralized parameter estimation. To accomplish this, a unique system model for Mm-Wave is suggested, in which the sensors process their annotations using digital and analog Precoders before transmitting them to the fusion center over a coherent channel. A two-stage hybrid precoding technique is discussed for the MIMO Mm-Wave system in this research. Mm-Wave technology has a lot of bandwidth, but it also has a lot of path, penetration, and absorption losses [24]. A low-complexity *orthogonal hybrid beamforming* (OHBF) approach is presented in this research. The orthogonal analog precoding matrix is generated using Householder reflectors. In the beam domain, it minimizes the size of the baseband Precoder as well as interuser interference. *Orthogonal Matching Pursuit*(OMP) and related algorithms are discussed in this paper. An efficient hybrid Precoder design is presented for maximization of gain and reduction of complexity using the WMMSE technique. For this, manifold optimization and iteration-based algorithms have been used for partially connected structures [25, 26].

Because typical hybrid precoding design cannot be adjusted to meet multiple system parameters, such as the length of bitstreams, hardware flexibility and efficiency are limited. In terms of system, technique, and architecture, this article describes parallel data-stream processing for the design of flexible and low-complexity hybrid Precoder [27–29]. The proposed approach was created with the goal of achieving adjustable precoding architecture by avoiding signal dependence between data streams. The proposed algorithm's performance and complexity were also simulated and evaluated in-depth in this article [15]. Algorithms were extended from narrowband system to wideband system for the maximization of spectral efficiency [11]. An algorithm that accurately approximates optimal unconstrained Precoders is designed using the basis pursuit principle that makes the use of low-cost RF hardware [9]. HP design for multiuser system was also studied in [13] and [14].

3. System Model

Let us consider a narrowband Mm-Wave massive MIMO system with hybrid precoding as shown in Figure 1. A digital Precoder A_{D} sends $\mathrm{N_s}$ data streams that pass through the analog Precoder A_{RF}. Analog Precoder consists phase shifter for increasing the beamforming gain. This RF signal is then collected by transmit antenna $\mathrm{N_T}$. Equivalently, the signal transmitted from the baseband point of view is $\mathrm{x} = A_{\mathrm{RF}}A_{\mathrm{Ds}}$, where s is the signal vector for data stream $\mathrm{N_s}$. For the simplification, assuming the normalized maximum transmitted power constraint is set to tr $\left(A_{\mathrm{RF}}A_{\mathrm{D}}A_{\mathrm{D}}^{\mathrm{H}}A_{\mathrm{RF}}^{\mathrm{H}}\right) \leq 1$.

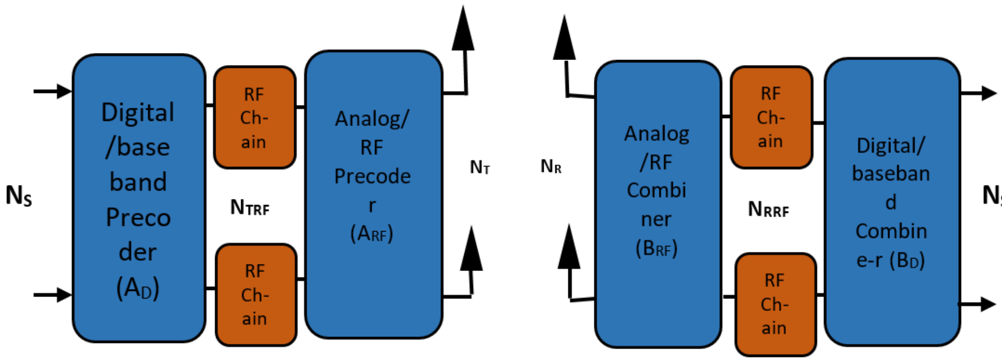

Fig. 1. Model of Millimeter-wave MIMO system.

Now, the baseband equivalent received signal at the receiver side can be written as $y = HAs + n$, where n is the Gaussian distributive additive white noise having '0' mean and σ^2 variance and H represents channel matrix of dimension $N_R \times N_T$ for frequency selective channel. channel matrix is given by:

$$H = \sqrt{\frac{N_T N_R}{N_c N_r}} \sum_{i=1}^{N_c} \sum_{j=1}^{N_r} \alpha_{ij} a_R \left(\theta_{ij}^R\right) a_T \left(\theta_{ij}^T\right)^H . \tag{1}$$

The geometric channel model of a millimeter-wave system is given with the help of N_c clusters each having N_r rays. α_{ij} denotes the complex gain of the i-th cluster with j-th ray. $a_R \left(\theta_{ij}^R\right)$ and $a_T \left(\theta_{ij}^T\right)$ are used for the representation of array vectors of the i-th cluster with j-th ray with an angle of arrival at receiving end and with the angle of departure at transmitting end. This response vector for a uniform planar array with half-wave spacing for N antennas can be expressed as

$$\mathrm{a}(\theta) = \frac{1}{\sqrt{N}} \left[1, e^{j\pi sin\theta} , \quad \ldots , e^{(N-1)\pi sin\theta}\right]^T$$

.

The signal received from the receiver side is firstly processed by analog combiner B_{RF} and then converted by digital combiner B_D to the baseband signal. The analog Precoder and combiner both are similar in nature having the same number of RF chains. Now the exact baseband signal after the hybrid combining process is given by:

$$y = B^H HAs + B^H n, \tag{2a}$$

$$y = B_D^H B_{RF}^H H A_{RF} A_D s + B_D^H B_{RF}^H n. \tag{2b}$$

The analog Precoder and combiner make use of phase shifters in their implementation. Therefore, the elements of A_{RF} and B_{RF} are subject to the unit modulus constraint such that $|A_{RF}| = 1$ and $|B_{RF}| = 1$.

$$R = \log_2 \left(\left| I_{N_s} + \frac{\rho}{N_s} (\sigma^2 B^H B)^{-1} B^H H A A^H H^H B \right| \right), \tag{3}$$

where σ^2 denotes noise power and I_{N_s} is identity matrix that is equal to $E = [ss^H]$. Also, $A = A_{RF} A_D$ and $B = B_{RF} B_D$.

4. Hybrid Precoding Optimization

This section describes MMSE expressions, Manifold Optimization algorithm and proposed MO-MMSE-based design conditions for hybrid Precoder.

4.1. MMSE–

The objective function for the optimization problem needed in the designing of hybrid Precoder and combiner is taken as modified MSE as a performance metric [12, 30] given by:

$$= E\left[\left(\alpha^{-1} y - s\right)\left(\alpha^{-1} y - s\right)^H\right], \tag{4}$$

where α is a scaling factor to be optimized jointly with hybrid Precoders [12].

By putting the value of y from equation 2(b) into 4(b), we get

$$= E\big[(\alpha^{-1}B_D^H B_{RF}^H H A_{RF} A_D s + B_D^H B_{RF}^H n - s) \\ (\alpha^{-1}B_D^H B_{RF}^H H A_{RF} A_D s + B_D^H B_{RF}^H n - s)^H\big]. \tag{5}$$

After some mathematical simplification, equation (5) becomes

$$MMSE = tr(\alpha^{-2}(B^H H A A^H H^H B - \alpha^{-1} B^H H A - \\ \alpha^{-1} A^H H^H B + \sigma^2 \alpha^{-2} B^H B + I_{N_s}). \tag{6}$$

Now, the optimization problem can be expressed with the derived MMSE equation in (6) as

$$\begin{aligned} &\min_{A_{RF}, A_D, B_{RF}, B_D, \alpha} MMSE \\ &\text{s.t.} \parallel A_{RF} A_D \parallel_F^2 \leq 1; \\ &\quad |[A_{RF}]_{ij}|^2 = 1, \forall i, j \\ &\quad |[B_{RF}]_{ml}|^2 = 1, \forall m, l. \end{aligned} \tag{7}$$

The solution of optimal B_D on fixing $A_{RF,}$ A_D, B_{RF} and B_{RF} is given by

$$B_D^{mmse} = (B_{RF}^H G G^H B_{RF} + \alpha I_{NTRF})^{-1} B_{RF}^H G, \tag{8a}$$

where $G = \alpha^{-1} H A$ and $\beta = \sigma^2 \alpha^{-2} N_T$.

By putting the optimal solution of B_D, the MMSE matrix (6) becomes

$$E^{MMSE} = \left(I_{N_s} + \beta^{-1} G^H B_{RF} B_{RF}^H G\right)^{-1}. \tag{8b}$$

Now, this theorem will be used in spectral efficiency maximization of hybrid Precoder.

4.2. Manifold Optimization Algorithm

The MO approach can be used to obtain a confined optimal A_{RF} when dealing with the constant modulus restriction [31, 32]. For this approach, first a Riemannian manifold is created for A_{RF} with the condition of constant modulus constraint and then update iteratively this optimization variable in the same direction as that of the Riemannian gradient. The derivation of the conjugate gradient to acquire the related Riemannian gradient in Euclidean space is the most difficult component of the application of the MO method [29, 33]. A conjugate gradient of the function gr(A_{RF}) with respect to A_{RF} is given by

$$gr(f\,(A_{RF})) = \text{gr}_{A_{RF}^*} f\,(A_{RF}) \odot P, \tag{9}$$

where P is the projection of gradient onto the tangent space of Riemannian manifold. After applying matrix differentiation property for the derivation of $\text{gr}_{A_{RF}^*} f(A_{RF})$ and some mathematical manipulation [34–36], we get:

$$gr(f(A_{RF})) = \frac{1}{\sigma^2 w}(A_{RF}(A_{RF}^H A_{RF})^{-1} A A_{RF}^H - I_{N_t}) \times \\ H^H B_{RF} B_D P^{-2} B_D^H B_{RF}^H H A_{RF} (A_{RF}^H A_{RF})^{-1}, \tag{10}$$

where $P = I_{N_s} + \frac{1}{\sigma^2 w} B_D^H B_{RF}^H H A_{RF} (A_{RF}^H A_{RF})^{-1} A_{RF}^H H^H B_{RF} B_D$ and $w = tr(B^H B)$.

The MO approach can be used to solve the optimization problem with constant modulus constraints using the derived Euclidean conjugate gradient [24]. The MO-HP algorithm is described in Algorithm 1. The following is a description of the detailed operation in the fourth stage. First, find the projection of the Euclidean gradient to get the Riemannian gradient onto the tangent space. Second, look for a point in tangent space. Then use the Armijo-Goldstein method along the Riemannian gradient. The step size is then determined by the condition. Last but not least, extract the results of the search lead back [30, 37, 38]. MO approach is used to deal with the phase shifter problems. Therefore, the optimization problem is considered in Riemannian space to update the analog Precoder (optimization variable) using some gradient descent algorithm.

Algorithm 1: The MO-HP algorithm
$Input : B_{RF}, B_D,\ H,\ \sigma^2\ \ Output :\ A_{RF}, A_D$
$1:$ Initialize $\mathbf{A}_{RF,0}$ with randomly elements for i $=\ 0;$
2: repeat
3: Compute $\nabla f(A_{RF,i})$ according to (10)
4: Update $A_{RF,(i+1)}$ with the help of MO method
5: i i+1;
6: until a stopping condition is satisfied.
Output: A_{RF} and A_D according to (12) and (13)

4.3. MO-MMSE based Hybrid Precoder

This sub-section emphasizes on the hybrid Precoder design in (7) by putting the receive combiner matrices B_D and B_{RF} as constant. As shown in [28, 33, 39], the original hybrid Precoder A_D can be considered as an unnormalized baseband Precoder. The optimization problem for Precoder can be formulated as

$$\begin{aligned}&\min_{A_{RF},A_D} tr(H_1^H A_{RF} A_D A_D^H A_{RF}^H H_1 - H_1^H A_{RF} A_D - A_D^H A_{RF}^H H_1 + \sigma^2 B^H B + I_{N_s})\\&\text{s.t. } tr(A_{RF} A_D A_D^H A_{RF}^H) \leq 1\\&|[A_{RF}]_{ij}| = 1, \forall i, j,\end{aligned} \tag{11}$$

where $H_1 = H^H B_{RF} B_D$ represents equivalent channel.

In this article, an optimization approach is to develop the optimal baseband precoding matrix A_D and by fixing A_{RF}, then a function of A_{RF} for the resulting objective, and then optimize A_{RF} by reducing the objective function with the constant modulus constraint. It can be verified that the optimal solution must be attained with the maximum transmitted power as a constraint.

Next, the solution in closed form for A_D rendering to the Karush-Kuhn-Tucker (KKT) conditions is given by

$$A_D = (A_{RF}^H H_1 H_1^H A_{RF} + \sigma^2 w A_{RF}^H A_{RF})^{-1} A_{RF}^H H_1. \tag{12}$$

Substituting the optimal A_D into (6) and after some mathematical manipulation, the resulting MMSE is given by

$$gr(f(A_{RF})) = tr((I_{N_s} + \frac{1}{\sigma^2 w} H_1^H A_{RF} (A_{RF}^H A_{RF})^{-1} A_{RF}^H H_1)^{-1}). \tag{13}$$

The optimization problem described in (6) is now reduced to the following one:

$$\min_{A_{RF}} \nabla f(A_{RF}) \quad \text{s.t. } |[A_{RF}]_{ij}| = 1, \forall i, j. \tag{14}$$

Here an algorithm based on Manifold Optimization is proposed for the optimization of the analog precoding matrix A_{RF} with the constant modulus constraint.

5. Simulation Results

The performance and existence of proposed MO-MMSE algorithm is tested in this section for a narrowband Mm-Wave MIMO system. Simulation results prove the performance comparison of proposed model with existing algorithms for different system configurations. Simulation parameters for described channel model are set in following manner: $N_c = 5; N_r = 10; N_T = 64; N_R = 64; N_s = 2$ and $4; N_{TRF} = N_{RRF} = 4$. These parameters are set by user. It is assumed that complex gain of the channel $\alpha_{ij} \sim \mathcal{CN}(0, 1)$. The angles generated in Laplacian distribution manner i.e. AOA and AOD are evenly and independently distributed in [0, 2π]. The angular spread is set to 100 in each cluster. The channel parameters are random in nature. It is also assumed that system synchronization and channel estimation is perfect. All simulations are performed on Matlab R2016b for 500 random channel samples.

The proposed algorithm provides higher spectral efficiency by optimizing the analog Precoder and digital Precoder parameters. These parameters are optimized using the Manifold optimization (MO) Hybrid precoding algorithm described in the previous section.

Figure 2 gives the performance in terms of BER for narrowband system with $N_s = 4$ and $N_{RF} = 4$. It shows the performance comparison of propose MO-MMSE algorithm with existing MO and OMP in [31] and [39].

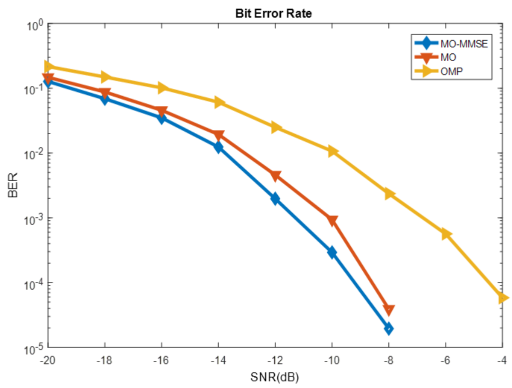

Fig. 2. BER Vs. SNR for Ns= 4.

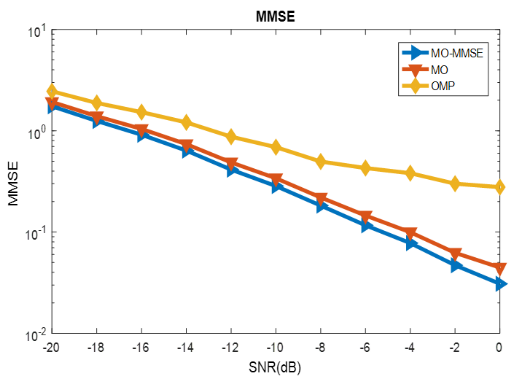

Fig. 3. MMSE Vs. SNR for Ns= 4.

From this comparison, it can be clearly seen that the proposed MO-MMSE algorithm outperforms the traditional OMP and MO algorithms. The reason behind the poorest performance of OMP algorithm is the reduced and limited number of sets of response vectors for antenna arrays. MMSE performance is also simulated for proposed MO-MMSE algorithm, existing MO algorithm and OMP algorithm for the above mentioned system with same configuration. From the simulaten results shown in Figure 3, it can be observed that again proposed work has superior performance against with MO and OMP algorithm.

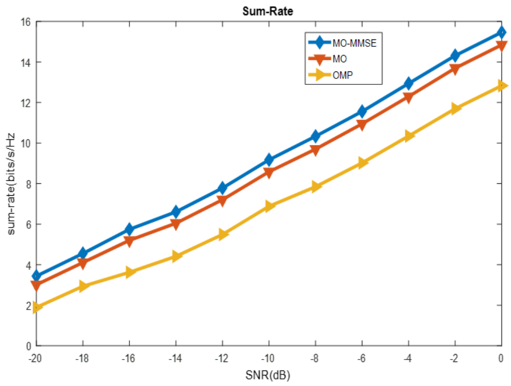

Fig. 4. Sum rate Vs. SNR for Ns= 2.

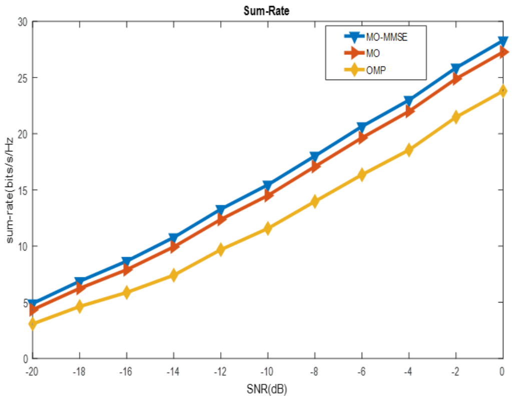

Fig. 5. Sum rate Vs. SNR for Ns= 4.

With increasing SNR, Figure 4 displays the spectral efficiency of several precoding techniques. Figure 4 shows that, when compared to other sub-connected-based approaches, the proposed algorithm has the maximum spectral efficiency. It is similar to the full-connected-based OMP method in terms of SNR. The OMP technique is based on a fully-connected structure that can generate more beamforming gain from all antennas than a sub-connected structure, but it also consumes more energy due to the use of a lot of phase shifters. Figure 5 also shows the spectral efficiency for Ns=4 with above-mentioned parameters. This figure clearly demonstrates that spectral efficiency goes on increasing with increase in signal to noise ratio. To validate the generality of the proposed algorithms, consider the system setup keeping $N_{TRF} = N_{RRF} = Ns$. For $N_{TRF} = N_{RRF} = Ns = 4$, the proposed MO-MMSE algorithms still outperform the baselines given in [8] and [9]. Figures 4 and 5 illustrate that the proposed MO-MMSE-based design provides a better sum rate as compared to MO and OMP algorithms.

Table 1. Comparison of Proposed Algorithm with existing MO and OMP algorithm

Algorithm	$N_c = 5; N_r = 10; N_T = 64; N_R = 64; N_S = 4; N_{TRF} = N_{RRF} = 4; SNR = -10$ dB		
	Bit Error Rate	MMSE	Spectral Efficiency (bps/Hz)
OMP	10^{-2}	10^{0}	11
MO	10^{-3}	$10^{-0.6}$	14
Proposed	$10^{-3.5}$	$10^{-0.4}$	16

Figures 6 and 7 show the variation of sum rate as a function of number of RF chain for two different configuration of system. These figures clearly illustrate that the performance sum rate increases in accordance with upturn in number of RF chains.

From the above simulation results, it can be concluded that proposed algorithm has low value of bit error rate and minimum mean square error and high spectral efficiency as compared to existing algorithms. The comparison of proposed algorithm with existing algorithm MO and OMP is shown in Table 1. The proposed MO-MMSE algorithm has better performance as compared to mentioned state of art algorithm *i.e.* MO and OMP algorithm.

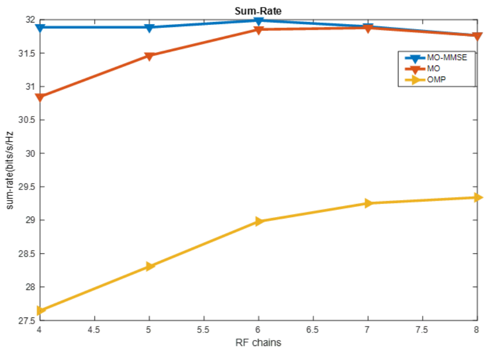

Fig. 6. Sum rate Vs. RF chains for 128*64 System.

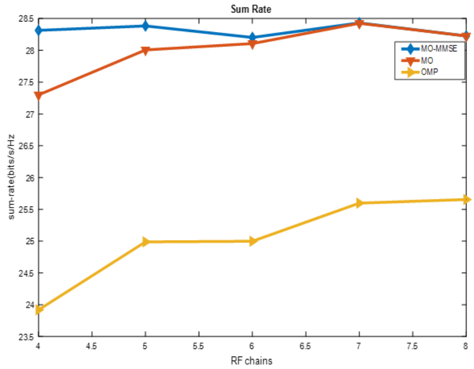

Fig. 7. Sum rate Vs. RF chains for 64*64 System.

6. Conclusion

In this article, an MO-MMSE based hybrid Precoder is designed for the maximization of spectral efficiency. For this, non-convex optimization problem is sub-divided into two different sub-problems. The proposed MO-MMSE algorithms has been applied for different configurations of MIMO system to attain substantial improvement over the existing ones. Simulation results for BER, MMSE and spectral efficiency showed that the proposed MO-MMSE based hybrid Precoder design approaches the optimal precoding with fewer RF chains, even though other algorithms balance the computational complexity and system performance. The proposed design can be extended to the multiuser system with other parameter optimization like minimization of BER with the different resolution of phase shifters.

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