ROMANIAN JOURNAL OF INFORMATION SCIENCE AND TECHNOLOGY
Volume **25**, Number 2, 2022, 133–149

Fuzzy AHP-based Prioritization of the Optimal Alternative of External Equity Financing for Start-ups of Lending Company in Uncertain Environment

Chun-Yueh LIN

Department of Public Finance and Tax Administration, National Taipei University of Business, 321, Sec. 1, Jinan Rd., Zhongzheng Dist., Taipei City 100, Taiwan
E-mail: ljy898@ntub.edu.tw

Abstract. This paper presents a novel model that applies a combination of triangular fuzzy numbers and the *analytic hierarchy process* (AHP) to the decision-making process to evaluate the optimal alternative of external equity financing for start-ups companies in the *financial technology* (FinTech) industry. First, this study develops the criteria and sub-criteria by modifying the Delphi method based on the literature review. Second, the fuzzy-AHP is used to obtain fuzzy weights of the criteria and sub-criteria. Third, the defuzzification method is used to rank the optimal alternative of the external equity financing for start-ups companies in terms of their overall weights on multiple evaluation criteria. Moreover, this study conducted a case study of lending start-ups in the FinTech industry to assess the optimal alternative of external equity financing. The results indicate that the equity crowdfunding is the optimal external equity financing alternative for lending start-ups in the FinTech industry and the costs of capital is the most important criterion for evaluating the optimal solution on an external equity financing approach. Academically, the fuzzy-AHP based decision-making theory can provide the decision-makers and administrators of the start-ups and entrepreneurs with valuable guidance for measuring the optimal alternative of external equity financing for start-ups companies in the FinTech industry. Commercially, the proposed processes which can provide support to decision-makers and entrepreneurs in start-ups companies as a valuable objective guide to measure the optimal external financing alternative.

Key-words: Fuzzy systems, external equity financing, peer to peer lending company, FinTech industry, start-ups, AHP.

1. Introduction

The *Pecking Order Theory* (POT) describes the sequence, as well as the alternatives, regarding the financing of start-ups. However, it is unable to account for the success of the financing if some factors come into play such as low funds, the corporations value, and the capability of financing. It does not work when the startup reaches the point where it has insufficient funds [1–3]. From the review of the existing literature, there is a wealth of evidence indicating that contrary to theoretical predictions, start-ups will first approach private equity investors before seeking external debt financing [4, 5]. Yet, Vaznyte & Andries (2019) demonstrated that it remains unclear why certain start-ups follow the traditional pecking order [6]. Past studies have demonstrated that the availability of external financing is seen as being more beneficial to the development of small firms (start-ups), which usually face insufficient funds [7, 8]. Furthermore, Walthoff-Borm *et al.* (2018) proposed three external financing alternatives: *angel financing* (AF), *equity crowdfunding* (EC), and *venture capital* (VC) [9].

In recent years, with the rise of *financial technology* (FinTech) in the global financial environment and implementation of Title III of the *Jumpstart Our Business Start-ups* (JOBS) Act in the United States, there has been an expansion in permissible equity crowdfunding [10]. Therefore, crowdfunding has emerged as a new relevant alternative for financing alongside more traditional means of financing new ventures [11]. In 2014, the worldwide crowdfunding volume saw an impressive year-over-year jump of 167% to reach US$ 16.2 billion [12]. Furthermore, Statista Inc. (2019) indicated that the total transaction value in the alternative financing segment amounted to US$ 11.7 million in 2019. The total transaction value is expected to show an annual growth rate of 14.3% between 2019 and 2023, resulting in a total value of US$ 20,000.2 million by 2023 [13]. Crowdfunding is this market's largest segment, with a total transaction value of US$ 6.9 billion in 2019. Thus, the crowdfunding market has grown rapidly during this period. Crowdfunding has several different forms. (1) Donation-based crowdfunding involves collecting charitable funding in support of causes and projects. (2) Rewards-based crowdfunding involves investors receiving non-monetary rewards in exchange for their contribution. (3) Debt-based crowdfunding offers a credit contract between funders and fundraisers. (4) Equity-based crowdfunding offers an equity stake in the target company [14, 15].

Angel financing investors, are private individuals investing directly in unlisted companies in which they have no family connections. Their investments have a significant economic impact and contribute to the survival of start-up firms [16, 17]. Past studies have shown that the size of the total angel investor market in the United States was US$ 24.6 billion in 2015, and the investment pool resulted in the creation of 270, 200 new jobs. Further, new ventures that attract AF financing exhibit higher survival rates, increased growth, more financing, more successful exits, and more employees compared with firms that cannot attract financing [18, 19]. The role of AF is important and, in some cases, essential for young ventures that require external financing [17]. Thus, AF is also an important financing alternative for start-up enterprises.

Another external equity financing alternative is equity crowdfunding. EC has emerged as a new financing alternative and plays an increasingly important role in financing start-up firms, as it provides new opportunities for entrepreneurs to target a broader group of external equity investors [20, 21]. The transactions cost will decrease by the use of social media, and EC introduces new mechanisms for entrepreneurs to establish a reputation with investors [22]. EC allows the matching of demand and supply in early-stage financing across a wide geographical area [23]. Furthermore, as traditional early-stage financing tends to be relational, its range and impact are generally narrow and contain hidden biases [24]. Hence, EC is both an important alternative for start-up firms' external equity financing and an early-stage financing alternative for new enterprises [9, 25].

Venture capital companies are a third equity financing alternative. Their critical activities comprise investment, support, exit, and reinvestment [26]. Generally, VC companies are equity holders that, in most instances, participate on the investee companies' boards of directors. They have the ability to guide and influence managerial decisions in terms of structure, operating procedure, and exit routes, including the decision to go public [27]. VC companies, as sources of external equity financing, positively affect firm performance by conducting post-investment monitoring and providing value-added services [28, 29]. Existing evidence demonstrates that businesses receiving VC achieve significantly higher employment growth rates [30, 31], higher profitability and R&D investment [32], and superior post-issue operating performance than those that receive no VC at all [33]. Consequently, VC is also an important financing alternative in start-up enterprises' external financing activity.

However, past studies do not provide theoretical insights or empirical evidence on how entrepreneurs or start-ups evaluate the optimal external equity financing alternative when choosing between crowdfunding and traditional external equity financing. In addition, the pecking order theory does not recognize how entrepreneurs choose between different sources of external equity financing, such as EC, AF, and VC [9]. The meaning is that start-ups or entrepreneurs of P2P lending companies for evaluating the optimal external equity financing alternative is a highly complex decision problem. Also, due to the strengths and weaknesses are different in external equity financing alternatives (EC, AF, and VC) which implicit the characteristics of nonlinear and ambiguous in this decision issue. With regard to the "optimization" issue that has some applications such as Beldjilali, *et al.* (2020) applied the optimization algorithm to construct a system for resolving vehicle routing problems [34]. They indicated that the system can make it possible for users to identify positions and track their vehicle fleet remotely via the web on any type of device. Bojan-Dragos, *et al.* (2021) used the grey wolf optimizer algorithm to develop the optimal fuzzy controllers for nonlinear issues to enhance the control system performance in electromagnetic actuated clutch systems [35]. The outcomes indicated that complex processes can be improved efficiently by type-2 fuzzy controller. Pozna, *et al.* (2022) presented a hybrid metaheuristic optimization algorithm which combines particle filter and particle swarm optimization algorithms and implemented it for the optimal tuning of proportional-integral-fuzzy controllers [36]. The results showed that this combination can reduce the energy consumption of the fuzzy control system. The aim of this study was to use multi-criteria decision-making to evaluate the optimal financing alternative for start-ups companies. The optimal concept in this study was different from the traditional optimization field (the cost/benefit function). Therefore, the optimal alternative concept of this work was concentrated on determining or obtaining suitable external equity financing targets for start-ups companies. Due to start-ups being too young or the financial structure is no good, external equity financing alternatives represent a potential target for start-ups to obtain capital. Based on the above, evaluating the optimal alternative selection was the core concept of this research. Evaluation of the optimal alternative is a *multi-criteria decision-making* (MCDM) issue that is generally solved by applying the *analytic hierarchy process* (AHP) method to obtain the weights of each criterion, the sub-criteria and the alternatives [37]. Many studies have employed AHP to construct a hierarchy model that can structure MCDM research issues [38–40]. Even though AHP is popular, this methodology cannot adequately resolve the inherent uncertainties and imprecisions associated with mapping the decision maker's perceptions into exact numbers [37, 41]. Hence, many previous works have combined the fuzzy set theory with AHP to solve uncertain and ambiguous issues [42–46]. Furthermore, the uncertain and vague expert opinions result in more complications, making it quite challenging to quantitatively predict the given problems, as compared to the use of qualitative prediction [47, 48]. In such a case, the fuzzy-AHP framework can assist in translating the qualitative expressions of human beings into meaningful numeric predictions [49]. Khan *et al.* (2019) indicated that the fuzzy-AHP framework has been utilized to rate human-opinion based optimal alternative selection problems [48]. Consequently, this study constructed a fuzzy-AHP based evaluation framework for fuzzy prioritization, in which expert comparison judgments are represented by fuzzy triangular numbers. Fuzzy-AHP was applied as the evaluation method in this study, and its effectiveness was illustrated by numerical examples. In addition to a literature review and survey of experts in the financial field, this study implemented the modified Delphi method and fuzzy-AHP to establish an evaluation process for estimating the optimal financing alternative for start-ups companies in the FinTech industry.

Therefore, this study integrated the fuzzy algorithm and AHP to construct an evaluation model for assessing the synthetic utility values of the criteria and sub-criteria of the optimal external equity financing alternatives, and then assigned a suitable relative weight to each criterion within the fuzzy hierarchical framework to rank the optimal external financing alternative. Academically, the fuzzy-AHP based decision-making theory could provide the entrepreneurs and the decision makers and administrators of start-ups s with valuable guidance for measuring the optimal external financing alternative start-ups of companies in the FinTech industry. Commercially, the proposed evaluation model could provide support to decision-makers and entrepreneurs of start-ups companies as a valuable objective guide to measure the optimal external financing alternative.

2. Evaluation model

The modified Delphi method was utilized to collect expert opinions, recognize the determinants of the evaluation framework, and obtain the weighted criteria and ranking using fuzzy-AHP. This evaluation model was based on the research of Lin (2020) and was used to solve the new research field and investigate such as new perspectives, sub-criteria, and different alternatives in the crowdfunding issue to obtain the optimal external financing alternative for start-ups companies [41]. The Delphi method, AHP algorithm, and fuzzy-AHP method are described in the following sections.

2.1. Delphi method

The Delphi method comprises several rounds of expert interviews regarding the inquiries, feedback, and arguments of previous rounds, during which the topics may change and the responses remain anonymous [50]. This method is especially suitable for explorative studies where changes in the relations between critical variables are intuitively expected, the respondents are geographically distant, and there is no dominating person in the discussion [51, 52]. This study implemented the Delphi method, and the results were statistically valid. The procedure was based on the works of Sung (2001), Wu, *et al.* (2007) and Lin (2020) [41, 53, 54].

2.2. AHP method

Saaty (1980) proposed the AHP model to solve complex decision problems [55]. This decision-making method deconstructs a complex MCDM problem into a hierarchy [56]. The AHP steps of this study were based on the study of Lin (2020) [41].

2.3. Fuzzy AHP method

The advantages of the developed fuzzy theory model facilitate its use in real-life situations for making effective decisions [57, 64]. Some study results have presented the combined application of the fuzzy set theory with AHP in areas such, performance evaluation, green supplier selection, Internet of Things evaluation, the impact of offshore outsourcing location prediction, shortest path problems, software selection, and reverse logistics evaluation [42, 46, 58, 59] Additionally, Kwak, *et al.* (2021) proposed a model to check the reductive property using the fuzzy modus ponens and fuzzy modus tollens in moving distance fields [60]. The results of the experiments showed that the proposed model is simple and effective for human thinking. Pozna and Precup (2014) developed a new framework implementing the fuzzy signatures rule with expert system modelling and applied the knowledge base and the data base as inputs to construct the expert systems [61]. The results illustrated that the algorithm is advantageous in the systematic and general formulation and allows for the building of uncertain expert systems. Hedrea, *et al.*, (2021) developed an architecture for the nonlinear concept of a *Tower CRrane* (TCR) system through the tensor product (TP)-based model transformation algorithm [62]. The results indicated that tested two scenarios and its performance of TP model which can provide the other dynamic systems modelling applications a valuable guideline.

In this study, *triangular fuzzy numbers* (TFNs) were used to present the preferences of one criterion over another according to the literature of Lin (2020) and Lu and Zhu (2018); however, the TFNs of this study represented a different application from that of Lu and Zhu (2018) [41, 63]. The structure of the TFNs is shown in Figure 1, and the membership function is shown in Table 1. This study employed fuzzy-AHP to conduct fuzzy hierarchical analysis by allowing TFNs for pair-wise comparisons and calculating the fuzzy preference-weights. The procedure of fuzzy-AHP algorithm was based on the work of Lin (2020) and the evaluation processes were as follows [41].

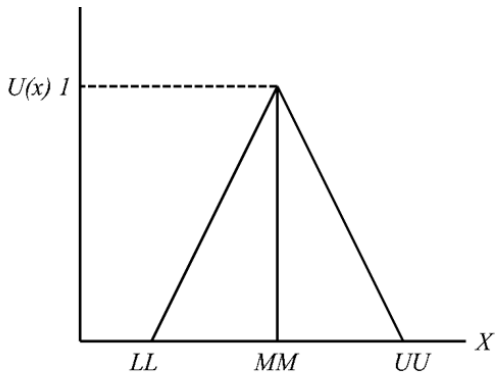

Fig. 1. Feature extraction process.

Table 1. Membership function of linguistic scale

Fuzzy number	Linguistic scale	Scaleof triangular fuzzy conversation	Scaleof triangular fuzzy reciprocal
9	Perfect	(8,9,9)	(1/9,1/9,1/8)
8	Absolute	(7,8,9)	(1/9,1/8,1/7)
7	Very good	(6,7,8)	(1/8,1/7,1/6)
6	Fairly good	(5,6,7)	(1/7,1/6,1/5)
5	Good	(4,5,6)	(1/6,1/5,1/4)
4	Preferable	(3,4,5)	(1/5,1/4,1/3)
3	Not bad	(2,3,4)	(1/4,1/3,1/2)
2	Weak advantage	(1,2,3)	(1/3,1/2,1)
1	Equal	(1,1,2)	(1,1,1)

Source: [80,81]

Step 1: Establish the problem and model

The problem should be clearly declared and decomposed into a rational system such as a network. The structure can be obtained according to the opinion of the decision makers through the modified Delphi method, brainstorming or other appropriate methods.

This study collected the sub-criteria, based on a literature review and expert interviews, and used a 7-point Likert scale for scoring, with answers ranging from very important (7) to very unimportant (1). After obtaining the scores, consistency testing was conducted using quartile deviation to sort the criteria.

Step 2: Establish the TFNs

As each number in the pair-wise comparison matrix indicates the subjective opinion of decision makers and is an ambiguous concept, fuzzy numbers work best for consolidating fragmented expert opinions through TFNs and the membership function of the linguistic scale (see Table 1). The TFNs $\widetilde{u}_{ij}$ can be obtained using the formula of Lin (2020) [41].

Step 3: Establish the fuzzy pair-wise comparison matrix

Saaty (1980) argued that the geometric mean accurately represents the consensus of experts, and it has become widely used in practical applications. In this study, the geometric mean method was used as the model for the TFNs [54].

According to expert opinion and linguistic scale transformation, construct the TFNs of $\widetilde{nu}_{ij}$ and develop the fuzzy pair-wise comparison matrix across $\widetilde{nu}_{ij}$. The definition of the fuzzy pair-wise comparison matrix can be obtained by the formula from Lin (2020) and include the $\widetilde{nu}_{ij}$, $\widetilde{LL}_{ij}$, $\widetilde{MM}_{ij}$ and $\widetilde{UU}_{ij}$ [41].

Step 4: Computation of the fuzzy weights

In the light of the fuzzy weights ($\widetilde{W}_i$) in Lin (2020) can be calculated [41].

Step 5: Defuzzification

The defuzzification process was according to the literature of Lin (2020). DF_i can be obtained by $\widetilde{W}_i$ [41].

Step 6: Normalization

As the summation of the defuzzification weights of each criterion is not equal to 1, it is necessary to normalize the defuzzification weights to a *new weight* (NW). NW_i refers to the weight of fuzzy-AHP in each criterion, which can be obtained by the literature of Lin (2020) [41].

Step 7: Rank the fuzzy-AHP weights

A set of alternatives can be ranked according to the descending order of NWi. The aim of this work was to integrate the fuzzy algorithm and AHP for assessing the synthetic utility values of the criteria and sub-criteria of the optimal external financing alternative and then, assign a suitable relative weight to each criterion within the fuzzy hierarchical framework to rank the optimal external financing alternative for start-ups companies in the Fintech industry. Section 3 presents an application of the proposed model using the above steps.

3. Empirical study

This study constructed indicators to evaluate the optimal external financing alternative for start-ups companies in the FinTech industry. The research framework is illustrated in Figure 2. An evaluation framework was constructed based on the modified Delphi method to assess the abovementioned optimal external financing alternatives using the fuzzy-AHP methodology. The proposed framework for evaluating the optimal external equity financing alternative comprise the following processes.

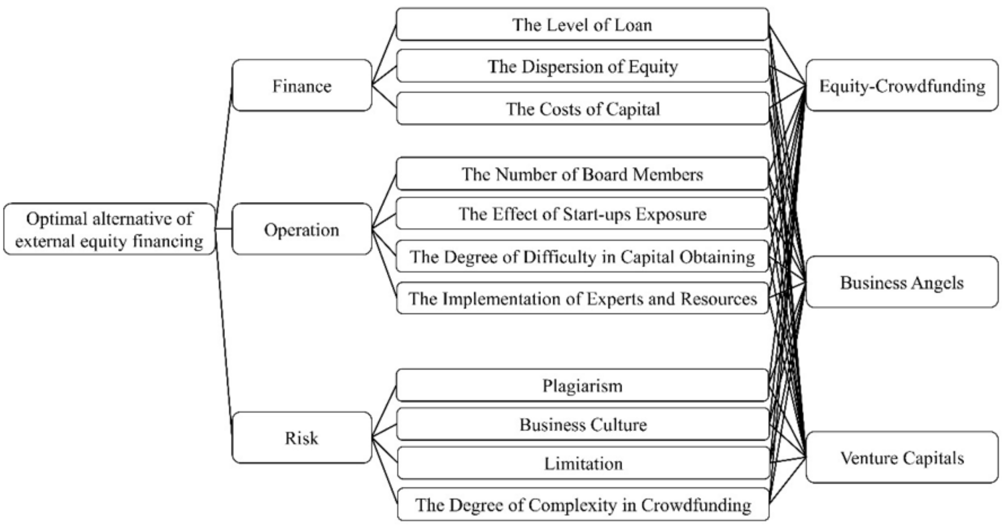

Fig. 2. The research framework of optimal alternative on external equity financing for lending company start-ups in the FinTech industry.

Step 1: Establish the problem and model

According to the literature review, a general consensus needed to be reached by the experts to establish a research model [65, 66]. Using the research model, this study obtained the final evaluation target of optimal external equity financing alternative for start-ups companies, along with three evaluation criteria, 11 sub-criteria, and the final external equity financing alternatives (Figure 2).

The evaluation criteria and sub-criteria that were applied to determine the optimal external equity financing alternative for start-ups companies in the FinTech industry are defined as follows:

1. Finance perspective:
   1.1. Level of loan [20, 67]: This refers to the level of loan from different external equity financing alternatives such as angel financing, equity crowdfunding, and venture capital.
   1.2. Dispersion of equity [20, 68, 69]: This refers to the dispersion of equity by the investor, such as that through angel financing, equity crowdfunding, and venture capital.
   1.3. Cost of capital: This refers to the cost of capital, fees, human capital, advertisement, interest rates, and so on from different external financing alternatives.
2. Operational perspective:
   2.1. Number of board members [68, 70]: This refers to the degree of intervention by the angel financing, equity crowdfunding, or venture capital investors in start-up firms.
   2.2. Effect of start-ups exposure [71, 72]: A start-up's reputation can be improved by using different external financing alternatives, such as angel financing, equity crowdfunding, and venture capital.
   2.3. Degree of difficulty in capital obtaining [67]: The degree of difficulty in obtaining capital is not consistent since the characteristics of the external financing alternatives are not the same.
   2.4. Implementation of experts and resources [73, 74]: This considers the investors' policies when using angel financing, equity crowdfunding, or venture capital during the implementation of experts and resources.
3. Risk perspective:
   3.1. Plagiarism [75]: A start-up's know-how or ideas can be plagiarized by an external crowdfunding platform.
   3.2. Business culture [76, 77]: Investors may affect a start-up's operational environment and culture, which could create conflict with the start-ups.
   3.3. Limitation [69, 78, 79]: Investors may intervene in a start-up's control and liquidation preferences and present terms restricting the founders when providing capital to the start-up.
   3.4. Degree of complexity in crowdfunding [67]: The degree of complexity is different when crowdfunding uses different external crowdfunding approaches. Therefore, the influence of the financing activity is very strong for start-up firms during early-stage financing.
4. External Equity Financing Alternatives
   4.1. Equity crowdfunding: Equity-based crowdfunding offers an equity stake in the target company.
   4.2. Angel financing: AF investors are private individuals who directly invest in unlisted companies in which they have no family connections. Their investments have a significant economic impact and contribute to the survival of the start-up firms.

4.3. Venture capital: VC firms are equity holders and, in most instances, participate on the investee companies' board of directors. Moreover, they have the ability to guide and influence managerial decisions in terms of the company's structure, operating procedure, and exit routes, including the decision to go public.

Step 2: Establish the TFNs

This study established the TFNs (see Figure 1) of each pair-wise comparison from the results of the AHP questionnaires through the fuzzy linguistic scale, and then, established the TFNs according to the literature of Lin (2020). Table 2 lists the results of the AHP questionnaire, which were used to obtain the pair-wise comparison matrix in the Tier 1. Next, this study implemented the fuzzy linguistic scale to calculate the fuzzy pair-wise comparison matrix (Table 3). The outcomes of the TFNs are shown in Table 2–7 and include Tier 1 to Tier 3.

Table 2. The pair-wise comparison matrix of the goal with criteria

Goal	Finance	Operation	Risk
Finance	1.000	4.217	1.587
Operation	0.237	1.000	0.275
Risk	0.630	0.636	1.000

Table 3. The fuzzy linguistic scale of the goal with criteria

Goal	Finance	Operation	Risk
Finance	(1.000 1.000 1.000)	(3.175 4.217 5.241)	(0.794 1.587 2.080)
Operation	(0.191 0.237 0.315)	(1.000 1.000 1.000)	(0.212 0.275 0.406)
Risk	(0.481 0.630 1.260)	(2.464 3.632 4.715)	(1.000 1.000 1.000)

Table 4. The pair-wise comparison matrix of finance criterion with sub-criteria

Finance	Level of loan	Dispersion of equity	Costs of capital
Level of loan	1.000	2.289	0.255
Dispersion of equity	0.437	1.000	0.193
Costs of capitalCosts of capital	3.922	5.181	1.000

Table 5. The fuzzy linguistic scale of finance criterion with sub-criteria

Finance	Level of loan	Dispersion of equity	Costs of capital
Level of loan	(1.000 1.000 1.000)	(1.260 2.289 3.302)	(0.203 0.255 0.347)
Dispersion of equity	(0.303 0.437 0.794)	(1.000 1.000 1.000)	(0.161 0.193 0.240)
Costs of capital	(2.886 3.916 4.934)	(4.159 5.191 6.213)	(1.000 1.000 1.000)

Step 3: Establish the fuzzy pair-wise comparison matrix The fuzzy pair-wise comparison matrix $(\widetilde{A})$ was determined by the literature of Lin (2020) [41]. The results of the level of "Goal" and criterion of "Finance" are as shown a sample. Table 8 shows the fuzzy pair-wise comparison matrix in the level of "Goal" and "Finance". Table 9 shows a sample of the results of the level of loan criterion in three alternatives.

Table 6. The pair-wise comparison matrix of sub-criterion of the level of loan with alternatives

Level of loan	Equity-crowdfunding	Angel fund	Venture capital
Equity-crowdfunding	1.000	2.884	3.780
Angel fund	0.347	1.000	2.289
Venture capital	0.265	0.437	1.000

Table 7. The fuzzy linguistic scale of sub-criterion of the level of loan with alternatives

Level of loan	Equity-crowdfunding	Angel fund	Venture capital
Equity-crowdfunding	(1.000 1.000 1.000)	(1.817 2.884 4.309)	(2.714 3.780 4.820)
Angel fund	(0.232 0.347 0.550)	(1.000 1.000 1.000)	(1.260 2.289 3.302)
Venture capital	(0.207 0.265 0.368)	(0.303 0.437 0.794)	(1.000 1.000 1.000)

Table 8. The fuzzy pair-wise comparison matrix of criteria and sub-criteria

Goal			
	Finance	Operation	Risk
Finance	1.000 1.000 1.000	3.175 4.217 5.241	0.794 1.587 2.080
Operation	0.191 0.237 0.315	1.000 1.000 1.000	0.212 0.275 0.406
Risk	0.481 0.630 1.260	2.464 3.632 4.715	1.000 1.000 1.000
Finance			
	Level of loan	Dispersion of equity	Costs of capital
Level of loan	1.000 1.000 1.000	1.260 2.289 3.302	0.203 0.255 0.347
Dispersion of equity	0.303 0.437 0.794	1.000 1.000 1.000	0.161 0.193 0.240
Costs of capital	2.886 3.916 4.934	4.159 5.191 6.213	1.000 1.000 1.000

Table 9. The fuzzy pair-wise comparison matrix of alternatives

Level of Loan			
	Equity-crowdfunding	Angel fund	Venture capital
Equity-crowdfunding	1.000 1.000 1.000	1.817 2.884 4.309	2.714 3.780 4.820
Angel fund	0.232 0.347 0.550	1.000 1.000 1.000	1.260 2.289 3.302
Venture capital	0.207 0.265 0.368	0.303 0.437 0.794	1.000 1.000 1.000

Step 4: Computation of the fuzzy weights The fuzzy weights (W_i) and real weights in this step were evaluated according to the research of Lin (2020) [41]. Table 10 shows a sample of the results for each tier of the fuzzy weights and real weights in the criteria, sub-criteria, and alternatives.

Step 5: Defuzzification

After computing the fuzzy weights, the defuzzification process was performed according the research of the Lin (2020) to estimate the real weights [41]. Defuzzification was performed on the samples of perspective of Finance and sub-criterion of level of loan to calculate the real weights (see Table 10).

Step 6: Normalization

The summation of the defuzzified weights was not equal to one in the same hierarchy; hence, the defuzzification weights were normalized by Eq. (1) to obtain the new weights, as follows: :

Table 10. The fuzzy weights and real weights of all criteria and sub-criteria

Goal	Fuzzy weights			Real Weights
	LL	MM	UU	
Finance	0.300	0.523	0.803	0.542
Operation	0.076	0.112	0.182	0.123
Risk	0.233	0.366	0.656	0.418
Finance	*LL*	*MM*	UU	
Level of loan	0.134	0.209	0.318	0.220
Dispersion of equity	0.077	0.109	0.175	0.120
Costs of capital	0.482	0.682	0.951	0.705
Level of loan	*LL*	*MM*	*UU*	
Equity-crowdfunding	0.367	0.611	0.995	0.658
Angel fund	0.143	0.255	0.442	0.280
Venture capital	0.086	0.134	0.240	0.153

$$Goal(NW_i) = \left| \begin{array}{c} Finance = \frac{0.542}{0.542+0.123+0.418} = 0.501 \\ Opertion = \frac{0.123}{0.542+0.123+0.418} = 0.113 \\ Risk = \frac{0.418}{0.542+0.123+0.418} = 0.386 \end{array} \right. \quad (1)$$

The NW_i of all criteria and sub-criteria are shown in Table 11.

Step 7: Rank the fuzzy-AHP weights

The normalization of the fuzzy-AHP weights was used to rank the sequence of the results (see Table 12). The sequence of the alternatives of three external equity financing for start-ups companies in the Fintech industry was EC (0.482) $>$ AF (0.378) $>$ VC (0.140). On the other hand, the fuzzy-AHP value of the perspective and sub-criteria were as follows: finance (0.501); operation (0.113); risk (0.386); level of loan (0.211); dispersion of equity (0.115); costs of capital (0.674); number of board members (0.009); effect of start-ups exposure (0.031); degree of difficulty in capital obtaining (0.067); implementation of experts and resources (0.016); plagiarism (0.064); business culture (0.031); limitation (0.227); and degree of complexity in crowdfunding (0.129). The sequential weights of the three perspective and 11 sub-criteria were ranked as costs of capital $>$ limitation $>$ degree of complexity in crowdfunding $>$ level of loan $>$ degree of difficulty in capital obtaining $>$ plagiarism $>$ dispersion of equity $>$ effect of start-ups exposure $>$ business culture $>$ implementation of experts and resources $>$ number of board members.

Therefore, the rank of this study showed that the sequential weights of three sub-criteria were costs of capital $>$ limitation $>$ degree of complexity in crowdfunding, indicating that costs of capital is the most important criterion for assessing the optimal equity financing alternative for start-ups companies in the FinTech industry.

Table 11. The NW_i of criteria and sub-criteria

						Equity-crowdfunding		Angel fund		Venture capital	
	Real Weights	Normalized		Real Weights	Normalized	Real Weights	Normalized	Real Weights	Normalized	Real Weights	Normalized
Finance	0.542	0.501	Level of loan	0.220	0.211	0.658	0.603	0.280	0.257	0.153	0.141
			Dispersion of equity	0.120	0.115	0.265	0.244	0.636	0.584	0.187	0.173
			Costs of capital	0.705	0.674	0.674	0.632	0.252	0.236	0.141	0.132
			Number of board members	0.077	0.078	0.388	0.366	0.536	0.507	0.134	0.127
			Effect of start-ups exposure	0.270	0.275	0.614	0.578	0.311	0.293	0.137	0.129
			Degree of difficulty in capital obtaining	0.577	0.587	0.645	0.597	0.273	0.253	0.163	0.150
			Implementation of experts and resources	0.136	0.138	0.296	0.277	0.640	0.598	0.134	0.125
			Plagiarism	0.154	0.167	0.598	0.533	0.338	0.301	0.186	0.166
			Business culture	0.073	0.081	0.318	0.296	0.623	0.578	0.136	0.126
			Limitation	0.541	0.585	0.667	0.639	0.233	0.223	0.144	0.138
			Degree of complexity in crowdfunding	0.307	0.334	0.558	0.513	0.383	0.352	0.147	0.135

Table 12. The fuzzy AHP synthesis value of criteria, sub-criteria and alternatives

Fuzzy AHP Synthesis Value												
Goal	Criteria	Local Weight	Global weight	Rank	Sub-criteria	Local Weight	Global weight	Rank	Alternatives	Local Weight	Global weight	Rank
G	Finance	0.542	0.501	1	Level of loan	0.220	0.211	4	EC	0.442	0.482	1
					Dispersion of equity	0.120	0.115	7	AF	0.346	0.378	2
					Costs of capital	0.705	0.674	1	VC	0.128	0.140	3
	Operation	0.123	0.113	3	Number of board members	0.078	0.009	11				
					Effect of start-ups exposure	0.275	0.031	8				
					Degree of difficulty in capital obtaining	0.587	0.067	5				
					Implementation of experts and resources	0.138	0.016	10				
	Risk	0.418	0.386	2	Plagiarism	0.167	0.064	6				
					Business culture	0.080	0.031	9				
					Limitation	0.587	0.227	2				
					Degree of complexity in crowdfunding	0.333	0.129	3				

4. Conclusions

In recent years, with the rise of FinTech in the global financial environment, external equity financing alternatives have emerged as a new relevant financing approach and have become popular for start-up firms and entrepreneurs. These external equity financing alternatives have different strengths and weaknesses, and their characteristics are not the same. It represents that start-ups or entrepreneur to evaluate the optimal external equity financing alternative is a highly complex decision problem. Also, the strengths and weaknesses are different in those external equity financing categories which implicit the characteristics of nonlinear and ambiguous in this decision issue. Previous works do not provide theoretical insights or empirical evidence on how start-ups or entrepreneurs to evaluate the optimal external equity financing alternatives and resolve the question of ambiguity in the decision-making process. Therefore, this study developed a decision-making process for measuring the optimal external equity financing alternative for start-ups companies using the concept of ambiguity to reduce the uncertainty in the FinTech industry via the integration of fuzzy theory and AHP theory in the decision-making process. The results indicated the combined of the fuzzy AHP process could resolve the uncertainty and reduce the ambiguity in the decision-making process when determining the optimal external equity financing alternatives.

The results of this study showed that the sequential weights of three sub-criteria were costs of capital $>$ limitation $>$ degree of complexity in crowdfunding, indicating that costs of capital is the most important criterion for assessing the optimal equity financing alternative for start-ups companies in the FinTech industry. Costs of capital represent the cost of capital in financing for start-up companies, which have to consider the relative costs. Limitations represent the restrictions implemented by the investors providing capital to start-up firms to control the start-up's liquidation preferences, which reduce the start-up's control. Finally, the degree of complexity in crowdfunding refers to the complexity of implementing different external equity financing alternatives. EC was found to be the optimal external equity financing alternative for start-ups companies in the FinTech industry. This means when start-ups or entrepreneur in the FinTech industry implement external equity financing, they should focus on EC alternatives that can improve the financing efficiency for start-ups in the FinTech industry.

This study integrated the fuzzy algorithm and AHP to assess the synthetic utility values of the criteria and sub-criteria for P2P lending company start-ups on implementing external equity financing evaluations. Decision makers and entrepreneurs of lending company start-ups in the FinTech industry are often not equipped with objective decision-making processes and evaluation criteria in uncertain or ambiguous environments for determining the optimal external equity financing alternative when implementing external equity financing. In the academic view, the fuzzy-AHP based decision-making theory could provide the decision makers and administrators of start-ups and entrepreneurs with valuable guidance for measuring the optimal external financing alternative based on the uncertainty in the FinTech industry. In the commerce view, the proposed processes could provide support to decision-makers and entrepreneurs as a valuable objective tool to obtain the optimal external financing alternative.

Acknowledgements. Authors want to thank the editor and reviewers for their helpful comments and suggestions to improve the quality of this work.

References

[1] C. CASAMATTA and C. HARITCHABALET, *Dealing with venture capitalists: shopping around or exclusive negotiation*, Review of Finance **18**(5), pp. 1743–1773, 2013.

[2] X. P. CHEN, X. YAO and S. KOTHA, *Entrepreneur passion and preparedness in business plan presentations: a persuasion analysis of venture capitalists' funding decisions*, Academy of Management Journal **52**(1), pp. 199–214, 2009.

[3] D. KIRSCH, B. GOLDFARB and A. GERA, *Form or substance: the role of business plans in venture capital decision making*, Strategic Management Journal **30**(5), pp. 487–515, 2009.

[4] R. E. CARPENTER and B. C. PETERSEN, *Capital market imperfections, hightech investment, and new equity financing*, The Economic Journal **112**(477), pp. F54–F72, 2002.

[5] S. PAUL, G. WHITTAM and J. WYPER, *The pecking order hypothesis: does it apply to start-up firms?*, Journal of Small Business and Enterprise Development **14**(1), pp. 8–21, 2007.

[6] E. VAZNYTE and P. ANDRIES, *Entrepreneurial orientation and start-ups' external financing*, Journal of Business Venturing **34**(3), pp. 439–458, 2019.

[7] P. AGHION, T. FALLY and S. SCARPETTA, *Credit constraints as a barrier to the entry and post-entry growth of firms*, Economic policy **22**(52), pp. 732–779, 2007.

[8] L. GUISO, P. SAPIENZA and L. ZINGALES, *The role of social capital in financial development*, American Economic Review **94**(3), pp. 526–556, 2004.

[9] X. WALTHOFF-BORM, A. SCHWIENBACHER and T. VANACKER, *Equity crowdfunding: First resort or last resort?*, Journal of Business Venturing **33**(4), pp. 513–533, 2018.

[10] V. IVANOV and A. KNYAZEVA, *US securities-based crowdfunding under Title III of the JOBS* Act. DERA White paper, 2017. [Accessed July 15, 2019].

[11] M. ROSSI, *The new ways to raise capital: an exploratory study of crowdfunding*, International Journal of Financial Research **5**(2), pp. 8–18, 2014.

[12] MASSOLUTION 2015CF, *The Crowdfunding Industry Report*, http://www.smv.gob.pe/Biblioteca/temp/catalogacion/C8789.pdf , 2015. [Accessed July 9, 2019].

[13] STATISTA, INC., *Alternative financing report 2019*, https://www.statista.com/outlook/297/100/alternative-financing/worldwide, 2019. [Accessed July 9, 2019].

[14] L. PURO, J. E. TEICH, H. WALLENIUS and J. WALLENIUS, *Bidding strategies for real-life small loan auctions*, Decision Support Systems **51**(1), pp. 31–41, 2011.

[15] A. AGRAWAL, C. CATALINI and A. GOLDFARB, *Some simple economics of crowdfunding*, Innovation Policy and the Economy **14**(1), pp. 63–97, 2014.

[16] MASON C. M., HARRISON R. T., *The size of the informal venture capital market in the United Kingdom*, Small Business Economics **15**(2), pp. 137–148, 2000.

[17] K. C. COX, J. LORTIE and S. A. STEWART, *When to pray to the angels for funding: The seasonality of angel investing in new ventures*, Journal of Business Venturing Insights **7**, pp. 68–76, 2017.

[18] W. R. KERR, J. LERNER and A. SCHOAR, *The consequences of entrepreneurial finance: Evidence from angel financings*, The Review of Financial Studies **27**(1), pp. 20–55, 2011.

[19] J. R. BECKER-BLEASE and J. E. SOHL, *New venture legitimacy: the conditions for angel investors*, Small Business Economics **45**(4), pp. 735–749, 2015.

[20] G. K. AHLERS, D. CUMMING, C. GNTHER and D. SCHWEIZER, *Signaling in equity crowdfunding*, Entrepreneurship Theory and Practice **39**(4), pp. 955–980, 2015.

[21] G. BRUTON, S. KHAVUL, D. SIEGEL and M. WRIGHT, *New financial alternatives in seeding entrepreneurship: Microfinance, crowdfunding, and peertopeer innovations*, Entrepreneurship Theory and Practice **39**(1), pp. 9–26, 2015.

[22] S. ESTRIN and S. KHAVUL, *Equity crowdfunding and the socialization of entrepreneurial finance*, Academy of Management Proceedings **2016**(1), 13036, 2016.

[23] A. AGRAWAL, C. CATALINI and A. GOLDFARB, *The Geography of Crowdfunding (= National Bureau of Economic Research Working Paper Series Nr. 16820)*, Cambridge, MA, 2011.

[24] E. MOLLICK and A. ROBB, *Democratizing innovation and capital access: The role of crowdfunding*, California Management Review **58**(2), pp. 72–87, 2016.

[25] M. FENWICK, J. A. MCCAHERY and E. P. VERMEULEN, *Fintech and the financing of entrepreneurs: From crowdfunding to marketplace lending*, 2017. SSRN: https://papers.ssrn.com/sol3/papers.cfm?abstract_id=2967891.

[26] M. PURI and R. ZARUTSKIE, *On the life cycle dynamics of venturecapitaland nonventurecapitalfinanced firms*, The Journal of Finance **67**(6), pp. 2247–2293, 2012.

[27] A. BUCHNER, A. MOHAMED and N. WAGNER, *Are venture capital and buyout backed IPOs any different?*, Journal of International Financial Markets, Institutions and Money **60**, pp. 39–49, 2019.

[28] M. M. MÄKELÄ and M. V. MAULA, *Cross-border venture capital and new venture internationalization: An isomorphism perspective*, Venture Capital **7**(3), pp. 227–257, 2005.

[29] S. A. FERNHABER and P. P. MCDOUGALL-COVIN, *Venture capitalists as catalysts to new venture internationalization: the impact of their knowledge and reputation resources*, Entrepreneurship Theory and Practice **33**(1), pp. 277–295, 2009.

[30] A. DAVILA, G. FOSTER and M. GUPTA, *Venture capital financing and the growth of startup firms*, Journal of business venturing **18**(6), pp. 689–708, 2003.

[31] F. BERTONI, M. G. COLOMBO and L. GRILLI, *Venture capital financing and the growth of high-tech start-ups: Disentangling treatment from selection effects*, Research Policy **40**(7), pp. 1028–1043, 2011.

[32] D. GUO and K. JIANG, *Venture capital investment and the performance of entrepreneurial firms: Evidence from China*, Journal of Corporate Finance **22**, pp. 375–395, 2013.

[33] JAIN B. A., KINI O., *Venture capitalist participation and the postissue operating performance of IPO firms*, Managerial and Decision Economics **16**(6), pp. 593–606, 1995.

[34] B. BELDJILALI, B. BENADDA and Z. SADOUNI, *Vehicles circuits optimization by combining GPS/GSM information with metaheuristic algorithms*, Romanian Journal of Information Science and Technology **23**(T), pp. 5–17, 2020.

[35] C.-A. BOJAN-DRAGOS, R.-E. PRECUP, S. PREITL, R.-C. ROMAN, E.-L. HEDREA and A.-I. SZEDLAK-STINEAN, *GWO-based optimal tuning of type-1 and type-2 fuzzy controllers for electromagnetic actuated clutch systems*, IFAC-PapersOnLine **54**(4), pp. 189–194, 2021.

[36] C. POZNA, R.-E. Precup, E. HORVATH and E. M. PETRIU, *Hybrid particle filter-particle swarm pptimization algorithm and application to fuzzy controlled servo systems*, IEEE Transactions on Fuzzy Systems, DOI: 10.1109/TFUZZ.2022.3146986, 2022.

[37] H. Y. TSAI, C. W. CHANG and H. L. LIN, *Fuzzy hierarchy sensitive with Delphi method to evaluate hospital organization performance*, Expert Systems with Applications **37**(8), pp. 5533–5541, 2010.

[38] N. AGUILAR-RIVERA, *A framework for the analysis of socioeconomic and geographic sugarcane agro industry sustainability*, Socio-Economic Planning Sciences **66**, pp. 149–160, 2019.

[39] G. BAFFOE, *Exploring the utility of Analytic Hierarchy Process (AHP) in ranking livelihood activities for effective and sustainable rural development interventions in developing countries*, Evaluation and Program Planning **72**, pp. 197–204, 2019.

[40] A. CALABRESE, R. COSTA, N. LEVIALDI and T. MENICHINI, *Integrating sustainability into strategic decision-making: A fuzzy AHP method for the selection of relevant sustainability issues*, Technological Forecasting and Social Change **139**, pp. 155–168, 2019.

[41] C. Y. LIN, *Optimal core operation in supply chain finance ecosystem by integrating the fuzzy algorithm and hierarchical framework*, International Journal of Computational Intelligence Systems **13**(1), pp. 259–274, 2020.

[42] P. T. M. LY, W. H. LAI, C. W. HSU and F. Y. SHIH, *Fuzzy AHP analysis of Internet of Things (IoT) in enterprises*, Technological Forecasting and Social Change **136**, pp. 1–13, 2018.

[43] J. LIU, Q. LI and Y. WANG, *Risk analysis in ultra-deep scientific drilling project-A fuzzy synthetic evaluation approach*, International Journal of Project Management **31**, pp. 449–458, 2013.

[44] S. GUPTA, U. SONI and G. KUMAR, *Green supplier selection using multi-criterion decision making under fuzzy environment: A case study in automotive industry*, Computers and Industrial Engineering **136**, pp. 663–680, 2019.

[45] F. AHMED and K. KILIC, *Fuzzy analytic hierarchy process: a performance analysis of various algorithms*, Fuzzy Sets and Systems **362**, pp. 110–128, 2019.

[46] C. LÓPEZ and A. ISHIZAKA, *A hybrid FCM-AHP approach to predict impacts of offshore outsourcing location decisions on supply chain resilience*, Journal of Business Research **103**, pp. 495–507, 2019.

[47] M. B. AYHAN, *A fuzzy AHP approach for supplier selection problem: A case study in a Gear motor company*, arXiv preprint arXiv:1311.2886, 2013.

[48] A. A. KHAN, M. SHAMEEM, R. R. KUMAR, S. HUSSAIN and X. YAN, *Fuzzy AHP based prioritization and taxonomy of software process improvement success factors in global software development*, Applied Soft Computing **83**, 105648, 2019.

[49] Z. GÜNGÖR, G. SERHADLIOĞLU and S. E. KESEN, *A fuzzy AHP approach to personnel selection problem*, Applied Soft Computing **9**(2), pp. 641–646, 2009.

[50] H. A. LINSTONE and M. TUROFF, *The Delphi Method: Techniques and Applications*, Addison-Wesley, Reading, MA, 1975.

[51] P. TAPIO, *Disaggregative policy Delphi: using cluster analysis as a tool for systematic scenario formation*, Technological Forecasting and Social Change **70**(1), pp. 83–101, 2003.

[52] M. NOWACK, J. ENDRIKAT and E. TUENTHER, *Review of Delphi-based scenario studies: quality and design considerations*, Technological Forecasting and Social Change **78**(9), pp.1603–1615, 2011.

[53] W. C. SUNG, *Application of Delphi method, a qualitative and quantitative analysis, to the Healthcare Management*, Journal of Healthcare Management **2**(2), pp. 11–19, 2001.

[54] C. R. WU, C. T. LIN and H. C. CHEN, *Evaluating competitive advantage of the location for Taiwanese hospitals*, Journal of Information and Optimization Sciences **28**(5), pp. 841–868, 2007.

[55] T. L. SAATY, *The analytic hierarchy process*, McGraw Hill, New York, NY, 1980.

[56] T. L. SAATY, *How to mark a decision: the analytic hierarchy process*, European Journal of Operational Research **48**(1), pp. 9–26, 1990.

[57] F. T. S. CHAN and N. KUMAR, *Global supplier development considering risk factors using fuzzy extended AHP-based approach*, Omega **35**, pp. 417–431, 2007.

[58] A. DEY, R. PRADHAN, A. PAL and T. PAL, *A genetic algorithm for solving fuzzy shortest path problems with interval type-2 fuzzy arc lengths*, Malaysian Journal of Computer Science **31**(4), pp. 255–270, 2018.

[59] R. KUMAR, S. A. EDALATPANAH, S. JHA, S. BROUMI, R. SINGH and A. DEY, *A multi objective programming approach to solve integer valued neutrosophic shortest path problems*, Neutrosoph Sets System **24**, pp. 134–149, 2019.

[60] C. J. KWAK, K. C. RI, S. I. KWAK, K. J. KIM , U. S. RYU, O. C. KWON and N. H. KIM, *Fuzzy modus ponens and tollens based on moving distance in SISO fuzzy system*, Romanian Journal of Information Science and Technology **24**(3), pp. 257–283, 2021.

[61] C. POZNA and R.-E. PRECUP, *Applications of signatures to expert systems modelling*, Acta Polytechnica Hungarica **11**(2), pp. 21–39, 2014.

[62] E.-L. HEDREA, R.-E. PRECUP, R.-C. ROMAN and E. M. PETRIU, *Tensor productbased model transformation approach to tower crane systems modeling*, Asian Journal of Control **23**(3), pp. 1313–1323, 2021.

[63] M. H. LU and K. P. ZHU, *Improved analytic hierarchy process based on triangular fuzzy number*, Advances in Social Science, Education and Humanities Research (ASSEHR) **13**(1), pp. 314–317, 2018.

[64] T. S. LIOU and M. J. J. WANG, *Ranking fuzzy numbers with integral value*, Fuzzy Sets and Systems **50**(3), pp. 247–55, 1992.

[65] G. LINDEN, K. L. KRAEMER and J. DEDRICK, *Who captures value in a global innovation network? The case of Apples iPod*, Communications of The ACM **52**(3), pp. 140–144, 2009.

[66] J. ALI-YRKKO, P. ROUVINEN, T. SEPPALA dna P. YLA-ANTTILA, *Who captures value in global supply chains? Case Nokia N95 smartphone*, Journal of Industry, Competition and Trade **11**(3), pp. 263–278, 2011.

[67] V. BESSIÈRE, E. STEPHANY and P. WIRTZ, *Crowdfunding, business angels, and venture capital: New funding trajectories for start-ups?*, 2nd Emerging Trends in Entrepreneurial Finance Conference, pp. 1–26, 2018.

[68] A. SIGNORI and S. VISMARA, *Does success bring success? The post-offering lives of equity-crowdfunded firms*, Journal of Corporate Finance **50**, pp. 575–591, 2018.

[69] J. ROGAN and G. SARFATI, *Why German entrepreneurs seek equity via crowdfunding?*, Revista da Micro e Pequena Empresa **12**(2), pp. 23, 2018.

[70] S. S. TURAN, *Financial innovation-crowdfunding: friend or foe?*, Procedia-Social and Behavioral Sciences **195**, pp. 353–362, 2015.

[71] J. A. HAUGE and S. CHIMAHUSKY, *Are promises meaningless in an uncertain crowdfunding environment?*, Economic Inquiry **54**(3), pp. 1621–1630, 2016.

[72] A. PRESENZA, T. ABBATE, F. CESARONI and F. P. APPIO,*Enacting social crowdfunding business ecosystems: The case of the platform Meridonare*, Technological Forecasting and Social Change **143**, pp. 190–201, 2019.

[73] J. A. BAUM and B. S. SILVERMAN, *Picking winners or building them? Alliance, intellectual, and human capital as selection criteria in venture financing and performance of biotechnology startups*, Journal of Business Venturing **19**(3), pp. 411–436, 2004.

[74] M. KNOCKAERT, B. CLARYSSE and M. WRIGHT, *The extent and nature of heterogeneity of venture capital selection behavior in new technologybased firms*, R&d Management **40**(4), pp. 357–371, 2010.

[75] P. PAZOWSKI and W. CZUDEC, *Economic prospects and conditions of crowdfunding*, Management Knowledge and Learning International Conference, pp. 1079–1088, 2014.

[76] N. DAI and R. NAHATA, *Cultural differences and cross-border venture capital syndication*, Journal of International Business Studies **47**(2), pp. 140–169, 2016.

[77] R. NAHATA, S. HAZARIKA and K. TANDON, *Success in global venture capital investing: do institutional and cultural differences matter?*, Journal of Financial and Quantitative Analysis **49**(4), pp. 1039–1070, 2014.

[78] S. N. KAPLAN and P. STROMBERG, *Venture capitals as principals: contracting, screening, and monitoring*, American Economic Review **91**(2), pp. 426–430, 2001.

[79] M. DA RIN, T. HELLMANN and M. PURI, *A survey of venture capital research*, Handbook of the Economics of Finance **2**, pp. 573–648, 2013.

[80] Y. C. CHOU, C. C. SUN and H. Y. YEN, *Evaluating the criteria for Human Resource for Science and Technology (HRST) based on an integrated fuzzy AHP and fuzzy DEMATEL approach*, Applied Soft Computing **12**(1), pp. 64–71, 2012.

[81] S. H. LEE, *Using fuzzy AHP to develop intellectual capital evaluation model for assessing their performance contribution in a university*, Expert Systems with Applications **37**(7), pp. 4941–4947, 2010.