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Methods for Assessing the Stability of Conditionally Stable Circuits by Using Small-signal Simulations

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Abstract. The popular Bode stability criterion is not suitable for assessing the stability of feedback systems for which the phase characteristic of their loop gain, T(s), crosses the –180° horizontal more than once. This fact is illustrated by four simple examples of such circuits, that also help demonstrate an extension of the Bode criterion proposed in this paper: the stability of a sub-class of conditionally stable circuits – whose T(s) comprises neither RHP poles nor poles in the origin, and the module frequency characteristic of their T(s) crosses the 0dB horizontal only once – can be assessed by analyzing the phase characteristic of their loop gain. The relationship between the phase margin of these circuits and their frequency and step response is also analyzed. Finally, the paper introduces a three-step method for assessing the stability of all feedback systems, including the conditionally stable ones, by using small-signal analysis. It involves extracting the poles and zeros of T(s), then plotting the corresponding Nyquist contour by using MATLAB scripts developed for this purpose.

Key-words: Bode stability criterion, conditionally stable circuits, multiple critical frequencies, Nyquist contour, Nyquist stability criterion, pole-zero analysis.

1. Introduction

Stability analysis of closed loop systems has evolved considerably over time [1] due to the continuous quest for effective and user-friendly new methods. The most popular such methods are based on the Bode [2] and Nyquist [3] stability criteria.

The Bode criterion is easier to implement in practice [4] but it does not yield valid results for certain types of feedback circuits. The limitations of the Bode stability criterion can be highlighted by applying it to some conditionally stable feedback circuits, whose loop gain phase characteristic crosses the –180° horizontal several times, with the first crossing frequency, f_{-180°, lower than the unity gain frequency, f_{0dB}. Such circuits can be stable although the Bode stability criterion indicates otherwise. The conditionally stable circuits are not theoretical-only cases; in fact, for some applications such as feed-forward operational amplifiers and low-dropout voltage regulators, they allow for a more convenient positioning of poles and zeros than the conventional absolutely stable approach.

Several revisions and extensions have been proposed for the Bode stability criterion, but a general and easy-to-implement solution has not been reported yet. The revised Bode criterion introduced in [5] is invalid for open-loop transfer functions, T(s), with poles in the complex *right half plane* (RHP) [6]. This limitation is avoided by the *generalized Bode criterion* (GBC) proposed in [6] but it can be implemented only if the precise mathematical expression of the open loop transfer function can be derived. Moreover, the GBC is rather complex and more cumbersome to use than the well-known Nyquist criterion.

In general, assessing the stability of feedback systems by using small-signal analysis is time-effective and provides insight into possible design optimization. Several methods for deriving the loop gain by using small-signal analysis [7, 8] or measurements [9] have been proposed. The Nyquist criterion [3] provides a general approach to assess the stability of closed-loop systems, including conditionally stable feedback circuits and circuits whose open-loop gain comprises RHP poles. It requires the drawing of the Nyquist contour, which in turn requires a precise representation of T(s), especially regarding the number and location of the RHP poles. Thus, the Nyquist criterion remains difficult to apply in practice [6] and none of the IC design environments employed by the industry outputs the Nyquist contour.

This paper addresses some of the issues faced by designers that use small-signal analysis to assess the stability of feedback systems based on the Bode and Nyquist stability criteria.

It is organized as follows: four conditionally stable feedback circuits are analyzed in the first part of Section 2; the results demonstrate the limited applicability of the Bode stability criterion and provide the basis for introducing a simple-to-verify condition that has to be met by a sub-class of conditionally-stable circuits in order for the Bode criterion to yield valid results for them. The last part of Section 2 focuses on the relationships between the phase margin (PM) of such conditionally-stable circuits and their output voltage overshoot, gain peaking and minimum phase, similar to the one presented in [10] for circuits whose T(s) comprise only a pair of complex poles. In Section 3, a three-step method for assessing the stability of any conditionally stable circuit is presented, as an extension of the method proposed in [11]. Main conclusions resulting from this work are drawn in Section 4.

2. Applying the Bode stability criterion to conditionally stable circuits: limitations and proposals

2.1. A circuit whose T(s) has no RHP poles is stable despite having a negative Gain Margin

Figure 1 presents the first conditionally stable circuit analyzed in this Section: a unity-gain voltage amplifier implemented by closing a negative feedback loop around an OpAmp with two gain stages, modeled by the *voltage-controlled voltage sources* (VCVS) E0 and E1, with voltage gains $a_1(s)$ and $b_1(s)$, respectively. The loop gain, $T_1(s)$, of this circuit has the following expression [11]:

$$T_1(s) = \frac{10000(s+1)(s+1)}{\left(\frac{s}{0.1}+1\right)\left(\frac{s}{0.1}+1\right)\left(\frac{s}{0.1}+1\right)} \tag{1}$$

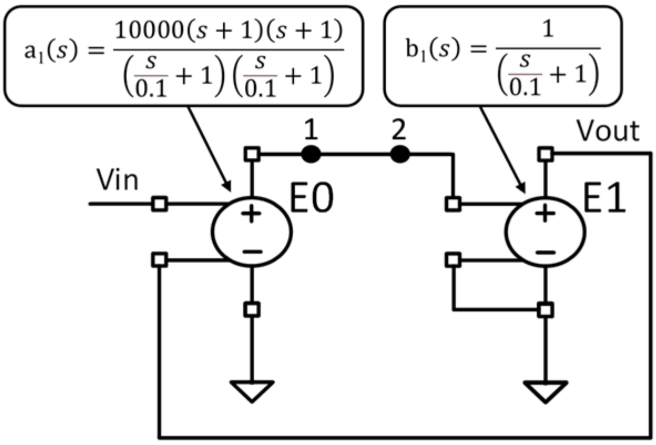

Fig. 1. A voltage buffer implemented by closing a negative feedback loop around an OpAmp with two gain stages, modelled by VCVS with gains $a_1(s)$ and $b_1(s)$.

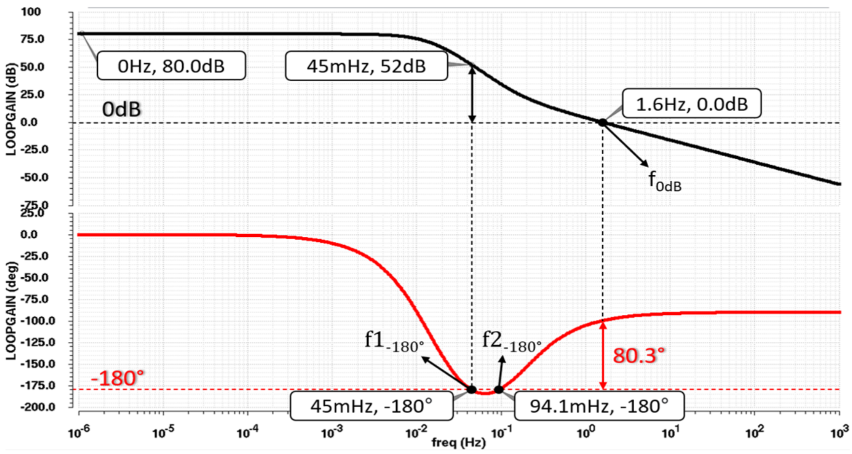

Fig. 2. Frequency characteristics of the $T_1(s)$ loop gain. One of the Bode stability metrics indicate instability: PM=80.38° but GM = –52dB.

Figure 2 presents the frequency characteristics of $T_1(s)$. One notices that the phase characteristic crosses the –180° horizontal line twice - at frequencies denoted $f1_{-180°}$ and $f2_{-180°}$ in Fig. 2 - before the $T_1(s)$ magnitude intersects the 0dB axis. By applying the Bode stability criterion one obtains a positive Phase Margin, PM = 80.38°, but a negative Gain Margin, GM = –52dB, and concludes that the circuit is unstable.

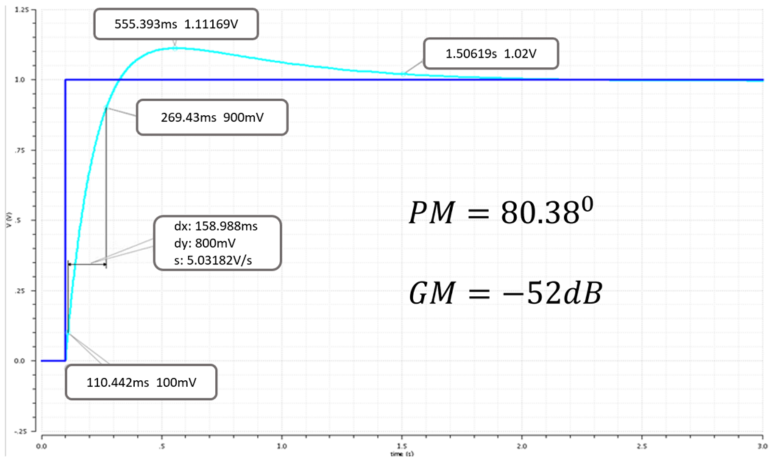

Fig. 3. The step response of the circuit shown in Fig. 1. It proves that the circuit is stable, despite having a negative gain-margin.

However, the step response of this circuit, presented in Fig. 3, is damped, without oscillations, irrefutably proving that the circuit is stable. To explain this discrepancy, the Nyquist criterion should be applied. This criterion states that the circuit is unstable if the Nyquist contour of its loop gain – $T_1(s)$ in this case – encircles counterclockwise the (–1, 0) point at least once [3].

The Nyquist contour of $T_1(s)$, shown in Fig. 4, exhibits two crossover frequencies. At first glance, it may appear that the Nyquist contour goes full circle around the (–1, 0) point, which would indicate an unstable system. To highlight that this is not the case, the half contour corresponding to the negative frequencies is drawn with a dashed line: it does not encircle the (–1, 0) point.

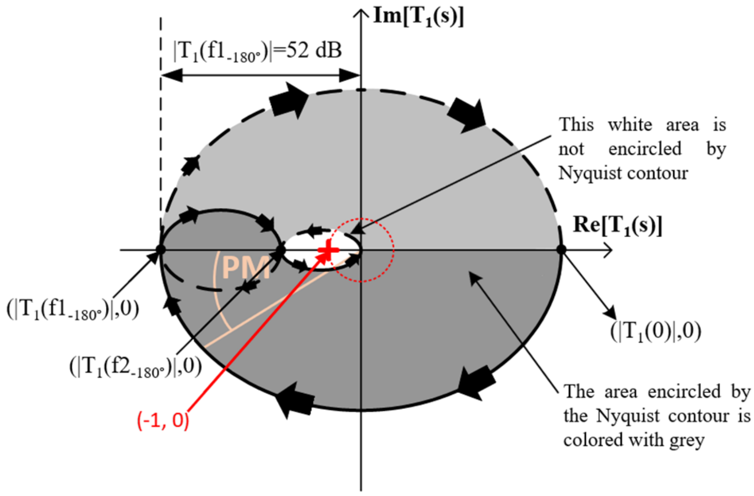

Fig. 4. The Nyquist contour of the circuit shown in Fig. 1 [11]. Note that the Nyquist contour does not encircle the (–1, 0) point.

The contour corresponding to the positive frequencies, drawn with continuous line, is a mirror image of the dashed line contour with respect to the real axis and it does not encircle the (–1, 0) point. The ellipse delimited by the origin and $(|T(f2_{-180^\circ})|, 0)$ that contains the (–1, 0) point is not encircled by the Nyquist contour, which means that the feedback circuit is stable.

The number of encirclements of the (–1, 0) point by the Nyquist contour can also be visually counted by using a vector with the origin in (–1, 0) pointing to, and moving clockwise along the full Nyquist contour, as shown in Fig. 5 [11]. The first 180° clockwise rotation of the vector, presented in Fig 5A, starts at point $(|T_0|, 0)$ and it ends at point $(|T(f2_{-180^\circ})|, 0)$. The second rotation, presented in Fig. 5B, starts at point $(|T(f2_{-180^\circ})|, 0)$ and ends in the same point where it started, with an angle of –360° (counter-clockwise rotation). The third and last rotation, presented in Fig. 5C, starts at point $(|T(f2_{-180^\circ})|, 0)$ and ends at $(|T_0|, 0)$ point, thus making a clockwise rotation of 180°. By adding up the three angles a net encirclement of zero degrees is obtained. This means that the Nyquist contour does not encircle the (–1, 0) point, confirming that the circuit is stable.

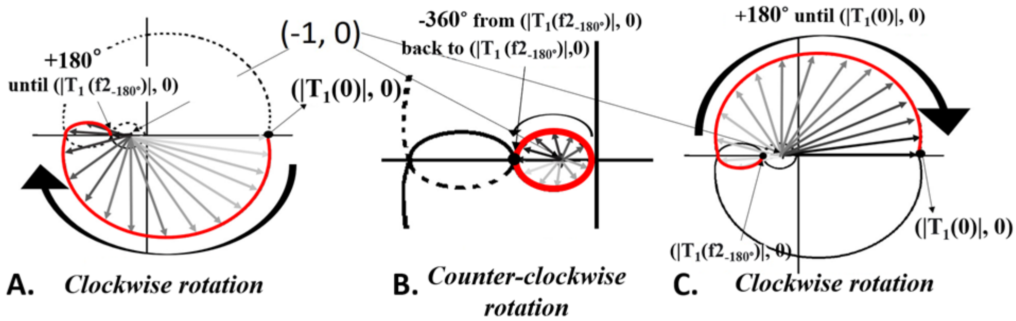

Fig. 5. Visual method for counting the number of encirclements of the (–1, 0) point the Nyquist contour presented in Fig. 4 [11].

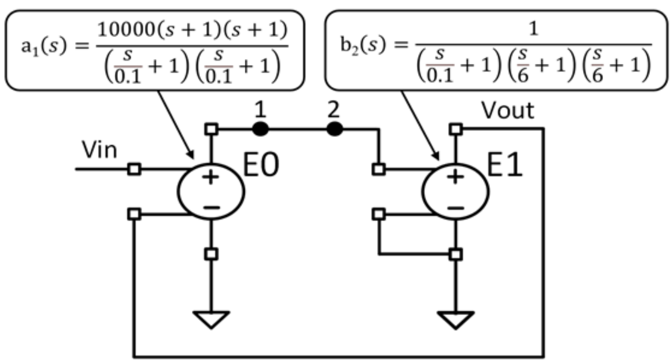

Fig. 6. A voltage buffer similar to the one shown in Fig.1 but with a different gain for the VCVS E1, that models the second gain stage of the OpAmp, b2(s).

2.2. The Bode stability criterion correctly predicts instability for a circuit whose T(s) has no RHP poles

Figure 6 present the second conditionally stable system analyzed in this paper. It is similar to the previous example but the gain of the VCVS that models the second gain stage comprises a double pole, placed at a frequency higher than the double zero introduced by the first stage. The resulting loop gain has the following expression [11]:

$$T_2\left(s\right)=\frac{10000\left(s+1\right)\left(s+1\right)}{\left(\frac{s}{0.1}+1\right)\left(\frac{s}{0.1}+1\right)\left(\frac{s}{0.1}+1\right)\left(\frac{s}{6}+1\right)\left(\frac{s}{6}+1\right)} \tag{2}$$

Figure 7 depicts the frequency characteristic of $T_2(s)$ phase. It crosses three times the –180° horizontal, with the third crossing frequency, $f3_{-180°}$, located before the unity-gain frequency, f_{0dB}. The Bode stability criterion indicates that the circuit is unstable as both the gain and phase stability margins are negative: GM = –55.4dB and PM = –13°. Fig. 8 shows the step response of this circuit; it confirms that the circuit is indeed unstable [11].

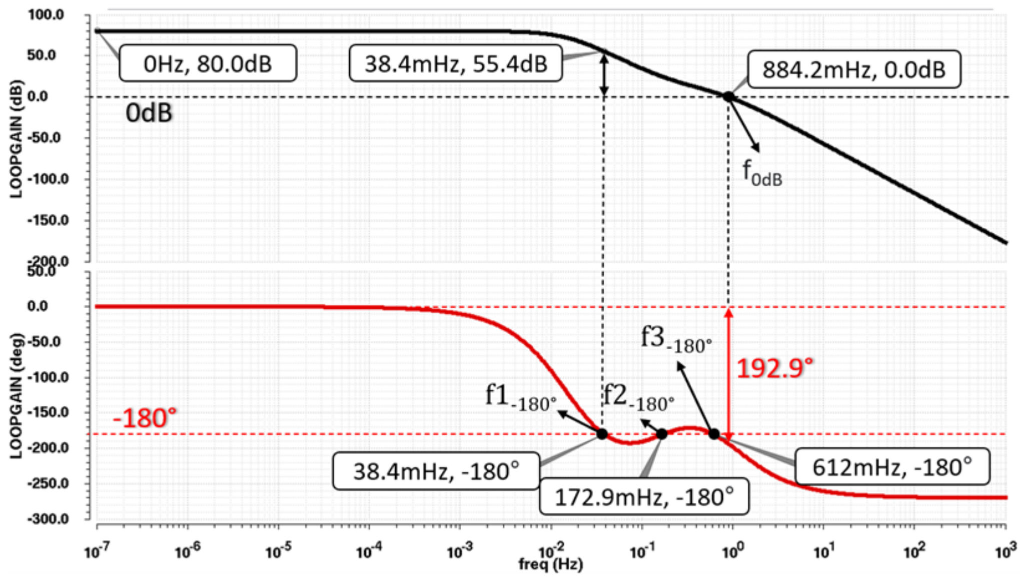

Fig. 7. Frequency characteristics of the $T_2(s)$ loop gain. Both Bode stability metrics indicate instability: PM = –12.9° and GM = –55.4dB.

The Nyquist contour of $T_2(s)$, presented in Fig. 9, has three crossover frequencies: $f1_{-180°}$, $f2_{-180°}$, $f3_{-180°}$ corresponding to $(|T(f1_{-180°})|, 0)$, $(|T(f2_{-180°})|, 0)$ and $(|T(f3_{-180°})|, 0)$, respectively, compared to the first example, where the Nyquist contour has only two crossover frequencies. For this example, there are two counter-clockwise encirclements of the (–1, 0) point, showing that the feedback circuit is unstable.

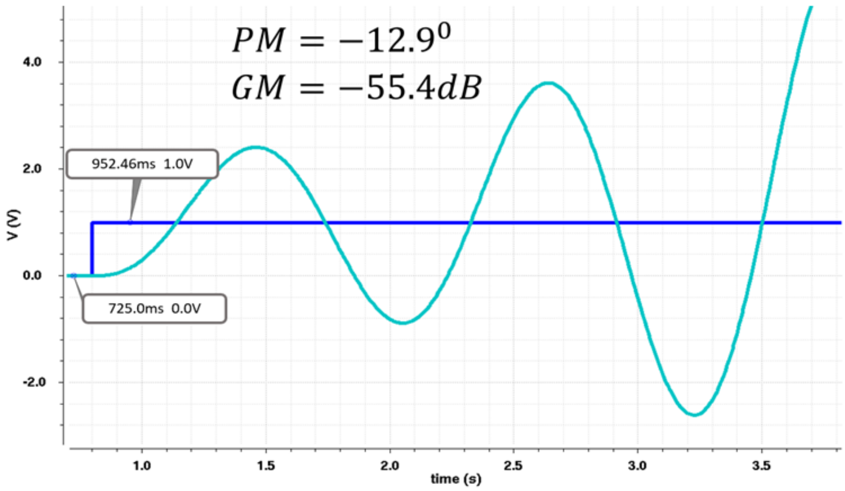

Fig. 8. The step response of the circuit shown in Fig. 6. It confirms that the circuit is unstable, as predicted by its negative phase margin.

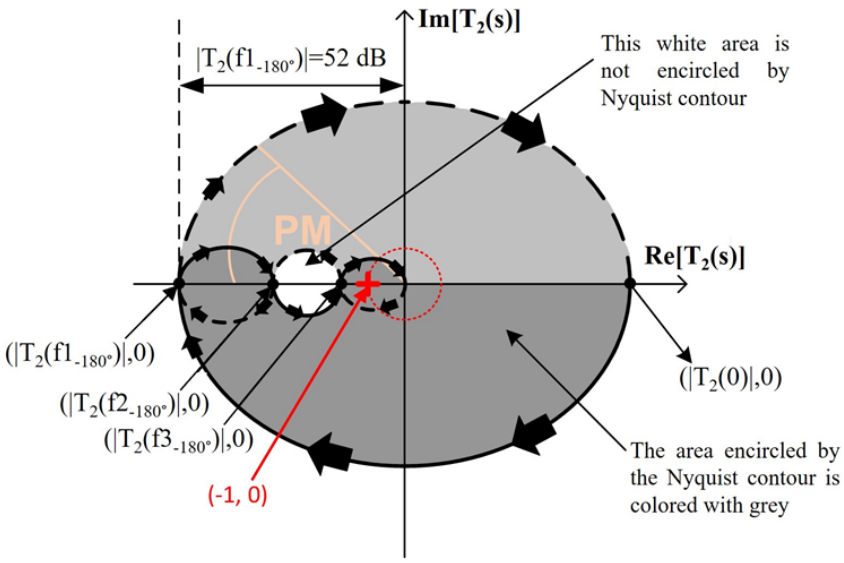

Fig. 9. Nyquist contour of the circuit presented in Fig. 6. The Nyquist contour does encircle the (–1, 0) point, thus predicting instability [11].

2.3. A circuit whose T(s) has poles in the origin and in the RHP is stable despite having a negative Gain Margin

Figure 10 presents the schematic of the third conditionally stable system analyzed here. It is similar to the previous examples, except for the expression of its open loop gain, $T_3(s)$ which has one pole in the origin and one pole in the RHP:

$$T_3(s) = \frac{16(s+1)}{s\left(\frac{s}{0.1}-1\right)\left(\frac{s}{5}+1\right)^2} \tag{3}$$

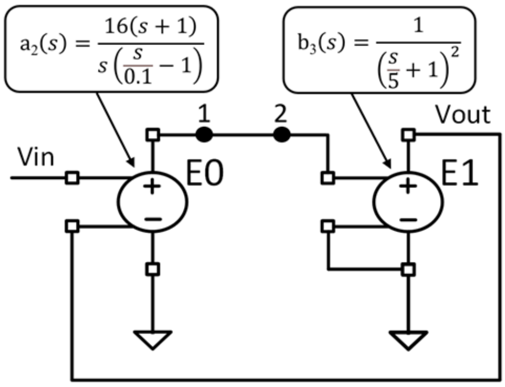

Fig. 10. A voltage buffer similar to the ones shown in Figs.1 and 6, but whose loop gain, $T_3(s) = a2(s) \cdot b3(s)$, comprise one pole in the origin and one pole in the RHP.

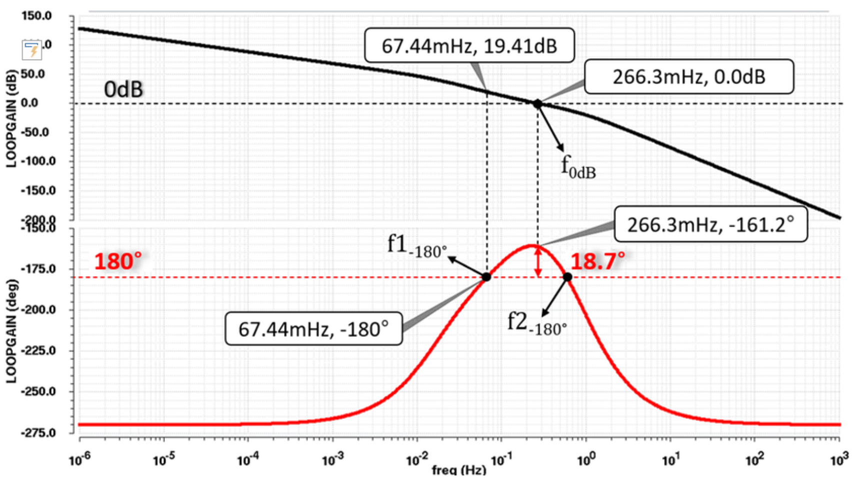

Fig. 11. Frequency characteristics of the $T_3(s)$ loop gain. The Bode stability criterion indicates instability, as the gain margin is negative: GM = –19.41dB.

Figure 11 presents the Bode pots of the $T_3(s)$ loop gain. This time, the phase characteristic stays above the –180° horizontal only in the vicinity of the unity gain frequency, f_{0dB}, which is flanked by the crossing points $f1_{-180°}$ and $f3_{-180°}$. By applying the Bode stability criterion one obtains a negative GM = –19.41dB measured at $f1_{-180°}$, leading to the conclusion that the circuit is unstable. However, the step response of this circuit exhibits only damped oscillations, proving that the circuit is in fact stable – see Fig. 12. For a more accurate stability analysis, let us apply the Nyquist criterion. The Nyquist contour of $T_3(s)$, shown in Fig. 13, encircles the critical point (–1, 0) once, in counter-clockwise direction; the encirclement is delimited by the points $(|T_3(f1_{-180°})|, 0)$ and $(|T_3(f2_{-180°})|, 0)$. According to the Nyquist criterion, the circuit shown in Fig. 10 is indeed stable, because the open loop gain $T_3(s)$ has one RHP pole and the corresponding Nyquist contour encircles the (–1, 0) point only once.

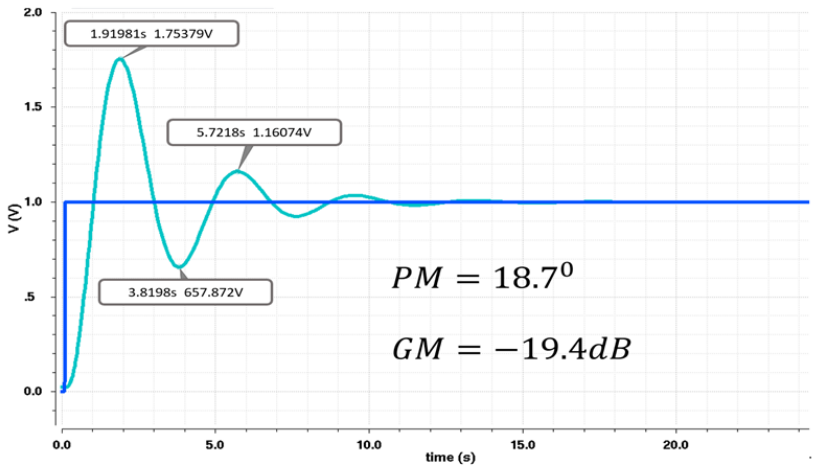

Fig. 12. The step response of the circuit shown in Fig. 10. It indicates that the circuit is stable, albeit not far from instability.

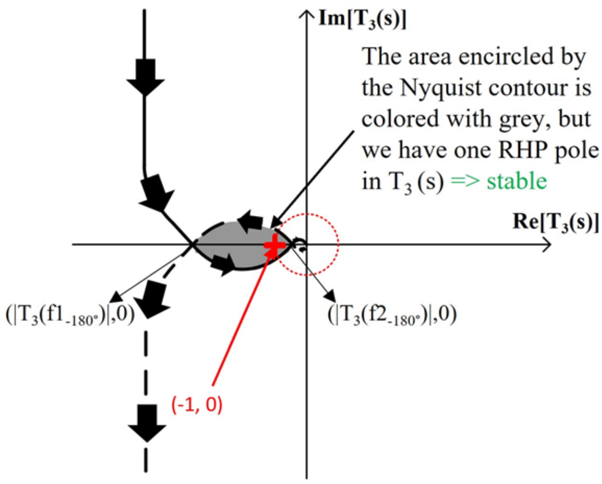

Fig. 13. Nyquist contour of the circuit presented in Fig. 10. The Nyquist contour does not encircle the (–1, 0) point, thus the circuit is stable [11].

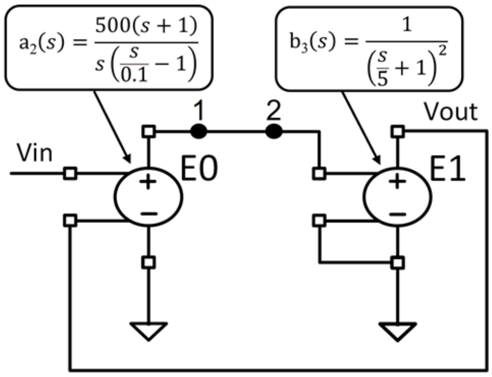

Fig. 14. A voltage buffer similar to the one shown in Fig. 10, but with a larger low-frequency gain.

2.4. The Bode criterion correctly predicts instability for a circuit whose T(s) has poles in the origin and in the RHP

Figure 14 presents the schematic of the fourth conditionally stable system analyzed here. It is similar to the circuit shown in Fig. 10, except for the larger low-frequency gain:

$$T_4(s) = \frac{500(s+1)}{s\left(\frac{s}{0.1}-1\right)\left(\frac{s}{5}+1\right)^2} \tag{4}$$

Figure 15 presents the Bode pots of the $T_4(s)$ loop gain. They have similar shapes with the frequency characteristics of $T_3(s)$ shown in Fig 11, but here the second crossing of the phase characteristic intersects the –180° horizontal before the unity gain frequency, $f2_{-180^\circ} <_{f0dB}$. The Bode stability criterion predicts instability, as one obtains negative values for both the gain and phase margins: GM = –49.31dB and PM = –43°.

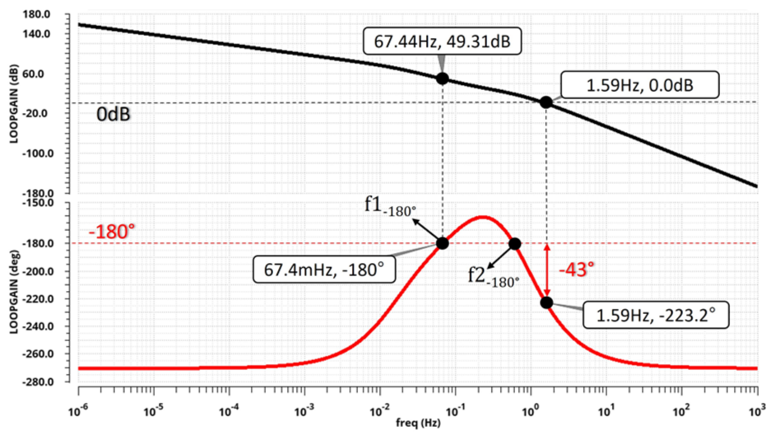

Fig. 15. Frequency characteristics of the $T_4(s)$ loop gain. The Bode stability criterion indicates instability, as both the gain and phase margins are negative: GM = –49.31dB, PM = –43°.

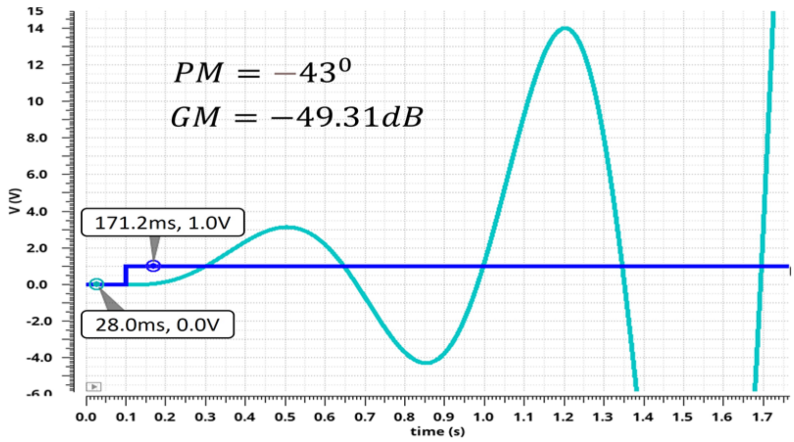

Fig. 16. The step response of the circuit shown in Fig. 14. It confirms that the circuit is unstable, as predicted by its negative phase and gain margins.

This is confirmed by the step response of this circuit shown in Fig. 16. The Nyquist contour of $T_4(s)$, shown in Fig. 17, also indicates that the circuit is unstable.

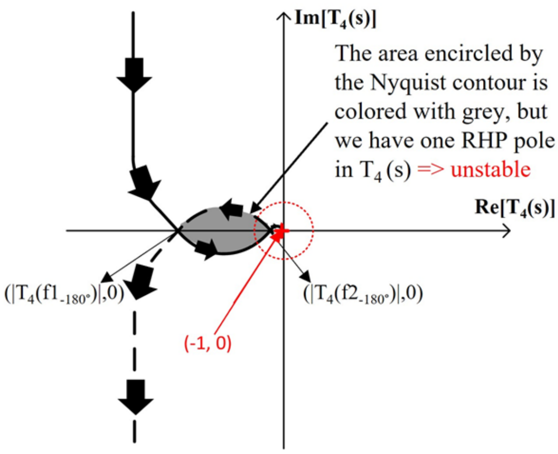

Fig. 17. Nyquist contour of the circuit shown in Fig. 14, with the loop gain $T_4(s)$ given by eq. (4). The Nyquist contour does encircle the (–1, 0) point, which predicts instability.

2.5. Cases the Bode stability criterion is not suited for

Table 1 summarizes the main results yielded by the circuit examples analyzed in the previous four sub-sections. Note that all four circuits have negative GM, so are unstable according to the Bode stability criterion. However, the stability of two of these circuits, which exhibit positive PM, has been proven by analyzing their step response, as well as by applying the Nyquist stability criterion.

It should be noted that these examples do not, in fact, contradict existing theory. First, in [2] Bode did not suggest that all conditionally stable were unstable, only warned against using them in circuits with cathodic tubes, because “the circuit may sing when the tubes begin to lose their gain because of age, and it may also sing, instead of behaving as it should, when the tube gain increases from zero as power is first applied to the circuit” [2]. Moreover, Bode stated that “conditional stability is usually regarded as undesirable” and the stability criterion “will consequently be restricted to absolutely” or “unconditionally” “stable amplifiers” [2]. Second, the fact that the gain and phase margins obtained from the Bode plots can be incorrect when assessing the stability of feedback circuits for which there are multiple frequencies at which their loop gain is equal to 1 or the phase is equal to -180° has been highlighted in textbooks such as [12]. In general, the gain margin and phase margin are well defined if the Nyquist curve intersects the negative real axis once, respectively if the Nyquist curve intersects the unit circle at only one point [12].

Similar analyses have been performed on numerous conditionally-stable circuits, leading to the conclusion that the Bode criteria is not suitable for assessing the stability of feedback systems for which the phase characteristic of their loop gain crosses the -180° horizontal more than once, but a positive Phase Margin remains a necessary condition for stability.

Table 1. Summary of stability-related parameters and features for the three conditionally-stable circuits analyzed in Sections II a, b, c and d.

Open loop transfer function T(s)	Number of poles in the origin within T(s)	Number of RHP poles within T(s)	Numberof times the phase characteristic crosses the –180° horizontal		GM [dB]	PM [deg]	Stability assessment yielded by the Bode criterion	Stability assessment yielded by the Nyquist criterion	Stability assessment based on the step response
			Total	Before f0dB					
$T_1(s)$	0	0	2	2	–52	80.3	unstable	stable	stable
$T_2(s)$	0	0	3	3	–55.4	–12.9	unstable	unstable	unstable
$T_3(s)$	1	1	2	1	–19.4	18.7	unstable	stable	stable
$T_4(s)$	1	1	2	2	–49.31	–43	unstable	unstable	unstable

2.6. Proposed extensions of the Bode criterion to systems with multiple –180°

The Bode stability criterion is easy to apply and has been embedded in most popular design tools. It yields valid results for large classes of feedback circuits but fails some conditionally-stable circuits, as shown in Table 1. Of those, the ones whose T(s) comprises no RHP poles are of most interest in practice.

Let us extend the Bode criterion to a sub-class of these circuits, that is, conditionally stable systems whose T(s) comprises neither RHP poles nor poles in the origin, and the module frequency characteristic of their T(s) crosses the 0dB horizontal only once:

- If the loop gain phase characteristic crosses the –180° horizontal line an even number of times before the unity gain frequency, the closed loop system is stable.
- If the loop gain phase characteristic crosses the –180° horizontal line an odd number of times before the unity gain frequency, the closed loop system is unstable.

The two situations above are illustrated by the circuits analyzed in Sections 2.a and 2.b, respectively: the Bode plots shown in Fig. 2 indicate that if the phase characteristic of T(s) crosses the –180° horizontal line an even number of times before f_{0dB} (as it is the case in Fig. 2, with two such crossings at $f1_{-180°}$ and $f2_{-180°}$) the phase must have a value larger than –180° at f_{0dB}, therefore the circuit has a positive PM. The Nyquist criterion confirms that such circuits are indeed stable, even if they have negative GM. Conversely, if the phase characteristic of T(s) crosses the –180° horizontal line an odd number of times before f_{0dB} – as is it the case in Fig. 7 - the circuit has a negative PM and it is unstable.

For circuits whose loop gains comprise poles in the origin and/or in the RHP, analyses performed in circuits similar to the ones presented in Sections 2.c. and 2.d, with the loop gains $T_3(s)$ and $T_4(s)$, indicate that a positive PM is necessary for stability if the loop gain phase characteristic crosses the –180° horizontal line an odd number of times before the unity gain frequency.

2.7. Relationship between the PM and frequency and step responses of conditionally stable circuits

Let us check whether the well-known relationships between the PM and the frequency and step responses of feedback circuits with loop gain that can be approximated by second-order all-pole functions described in [10] are valid for conditionally stable circuits as those presented in the previous sub-sections.

The step response of feedback circuits with loop gain that can be approximated by second-order all-pole functions, exhibits no Overshoot for PM $>$ 76.3°; the frequency response of such circuits has no Gain Peaking if PM $>$ 65.5° [10]. To examine if these conditions are valid for conditionally stable systems, simulations were run on circuits derived from the one analyzed in Section 2.a, shown in Fig. 1. PM values between 0° and 90° were obtained by changing the location of zeroes within $T_1(s)$, so that the phase characteristics of the resulting loop gain crossed the –180° horizontal line twice before the magnitude characteristics intersected the 0dB axis – that is, ensuring that the derived circuits remained stable. The behavior of these circuits was monitored through the step response and parameters related to closed loop frequency and step responses: overshoot and gain peaking. The minimum phase of the loop gain phase characteristic that is measured between the two points the phase crosses the –180° horizontal, $f1_{-180°}$ and $f2_{-180°}$, was also monitored.

Figure 18 presents the resulting graphs, that relate the overshoot, gain peaking and minimum phase to the phase margin. Fig. 19 presents the step response of three circuits derived from the one shown in Fig. 1, but with the zeros location modified so that their PM took the following values: 80.38°, 60° and 13.28°.

Figure 18 and Figures 19.A and 19.B show that the gain peak and the ringing decrease when the PM increase, somewhat similar to the relationships described in [10] for feedback circuits with loop gain that can be approximated by second-order all-pole functions. The differences lie in the values at which the ringing and gain peaking occur: the conditionally-stable circuit depicted in Fig. 19A exhibits an overshoot of about 11% and about 1dB of gain peaking, despite its 80.38 PM. The circuit depicted in Fig. 19.B has PM = 60°- the "typical" design target for unconditionally stable circuits – but its step response exhibits a 27% overshoot.

Figures 18A and 18B indicate that the PM should go up to 90° in order to get practically no gain peaking and no overshoot.

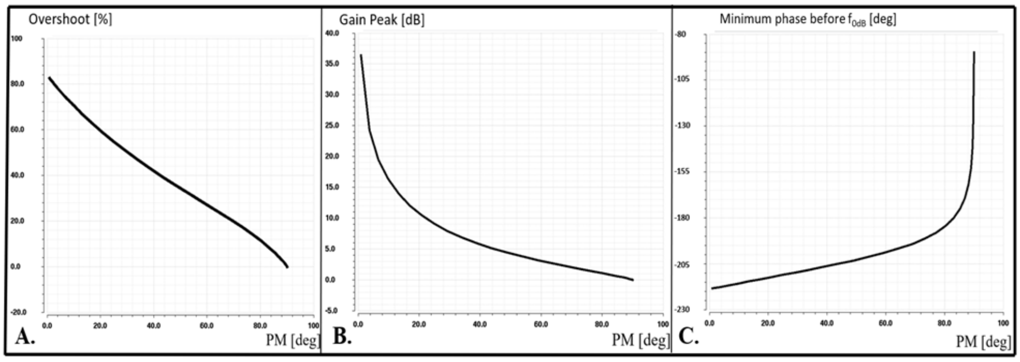

Fig. 18. The PM influence on a conditionally stable circuit behavior: A). Overshoot vs PM; B). Gain Peak vs PM; C). Minimum loop gain phase before f0dB *vs* PM.

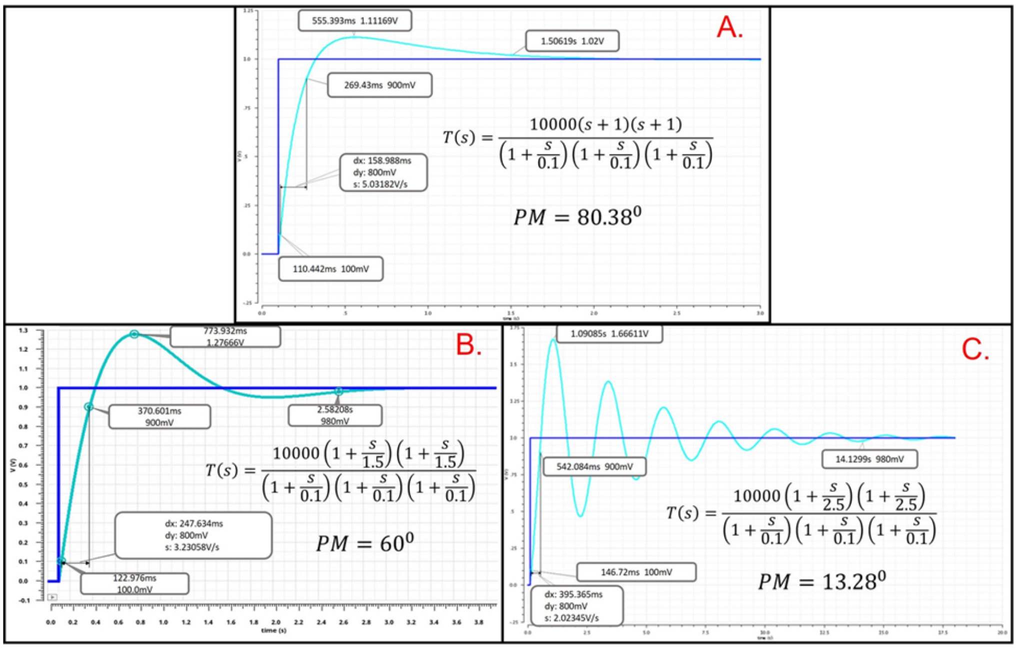

Fig. 19. Step response of conditionally stable circuits derived from the one shown in Fig. 1 but with the zeroes location modified so that their PM took the following values: A) 80.38°, B) 60° and C) 13.28°.

3. A practical method for assessing the stability of conditionally stable circuits by using small-signal analysis

This Section introduces a practical method for assessing the stability of feedback systems by using small-signal analysis, that can be applied to all conditionally stable circuits. The analysis performed in the previous Section demonstrated that, while the Bode criterion is not suitable to certain classes of conditionally stable circuit, the Nyquist criterion yields reliable results for all such circuits. Despite its advantages, the Nyquist criterion is seldom used in practice because the design environments most used within the industry do not provide the Nyquist contour. Furthermore, the methods based on the Nyquist contour for deriving popular stability metrics such as the GM and PM proposed so far [12] are far more difficult to implement and use then their Bode counterparts.

The Nyquist criterion states that a system is stable if the number of open-loop transfer function poles in the RHP is equal to the number of times that the Nyquist contour for this function makes anti-clockwise encirclements of the (–1, 0) point [3, 12]. Usually, applying this criterion implies the derivation of the loop gain analytical expression, T(s), then the identification of its poles and zeros. These steps are often difficult to perform for real-life circuits, and the whole approach is somewhat cumbersome for designers that rely on simulation-based analyses.

This section presents an effective method for deriving the Nyquist contour, starting from the results of the pole-zero analysis provided by most major IC design environments. It involves extracting the poles and zeros of the loop gain, determined through small-signal simulations, and plotting the corresponding Nyquist contour by using a MATLAB script developed for this purpose. The main steps of the proposed method are detailed below and exemplified on the four circuits analyzed in the previous Section.

3.1. Step 1: Find the loop gain poles and zeros

The loop gain, T(s), of a feedback circuit can be determined through small-signal simulations by breaking the feedback loop – for example between points denoted “1” and “2” in Figs. 1, 6, 10 and 14 - then applying one of the many well-known methods available for this [7, 8]. The direct approach – place a test voltage source with AC=1 at the “test” side of the loop breaking point, then measure the resulting voltage developed at the “measure” side – is suitable for the circuits analyzed in the previous Section, based on ideal voltage-controlled voltage sources, with infinitely large input impedances and zero output impedances. Fig. 20 presents the testbench used to derive the loop gain of the circuit shown in Fig. 1 through small-signal simulations. Similar testbenches have been used to derive the loop gains of the circuits shown in Figs. 6, 10 and 14. Table 2 lists the location of the poles, P, and zeros, Z, of the loop gains for the four circuits, yielded by small-signal pole-zero analysis of their loop gains.

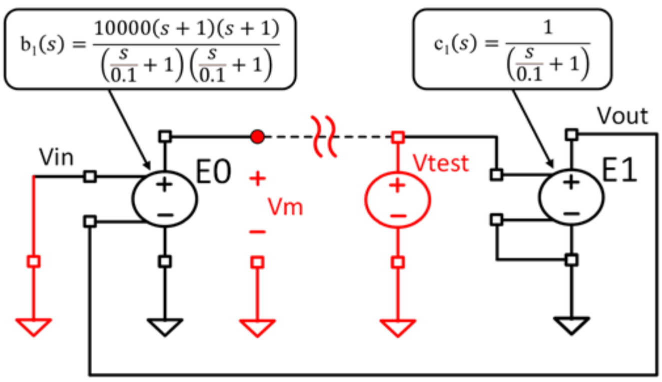

Fig. 20. Testbench for extracting the Poles, Zeros and DC Gain from $T_1(s)$.

Table 2. Loop gain parameters of circuits shown in Figs. 1, 6, 10 and 14: low-frequency gain, location of their poles and zeros

Circuit from	*Poles of T(s)*				
	P1 [Hz]	**P2 [Hz]**	**P3 [Hz]**	**P4 [Hz]**	**P5 [Hz]**
Ex. Fig. 1, $T_1(s)$	–15.9m	–15.9m	–15.9m	–	–
Ex. Fig. 6, $T_2(s)$	–15.9m	–15.9m	–15.9m	–955m	–955m
Ex. Fig. 10, $T_3(s)$	0	15.9m (RHP)	–795m	–795m	–
Ex. Fig. 14, $T_4(s)$	0	15.9m (RHP)	–795m	–795m	–
	Zeros of T(s)		–	–	*Gain@1Hz*
	Z1 [Hz]	**Z2 [Hz]**	–	–	
Ex. Fig. 1, $T_1(s)$	–159m	–159m	–	–	10k
Ex. Fig. 6, $T_2(s)$	–159m	–159m	–	–	10k
Ex. Fig. 10, $T_3(s)$	–159m	–	–	–	130dB
Ex. Fig. 14, $T_4(s)$	–159m	–	–	–	160dB

3.2. Step 2: Re-construct the T(s) frequency characteristics in MATLAB

The numerical values of the poles and zeroes yielded by the first step into the general expression are loaded into a MATLAB script, which outputs the expression of T(s) and its phase and magnitude frequency characteristics in MATLAB. The plotted phase and magnitude frequency characteristic are then compared with the Bode plots yielded by the small signal simulations run on the actual circuit.

This comparison is performed by a MATLAB script that compare point-by-point the two sets of characteristics (from SPICE and MATLAB). In Fig. 21, a few points on the Bode plots of $T_1(s)$ are compared as an example.

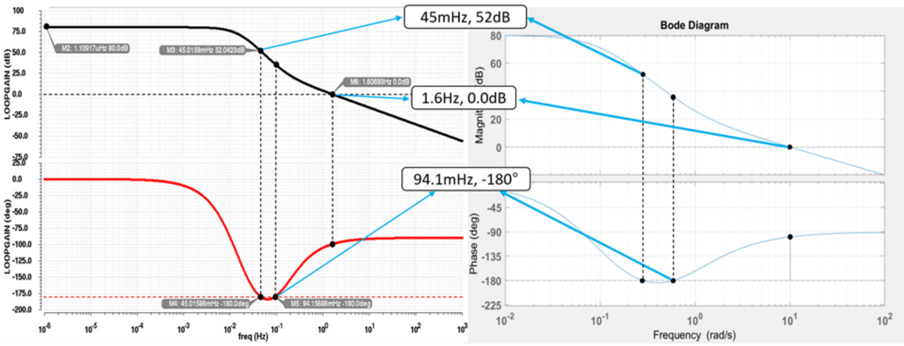

Fig. 21. Comparison of the Bode characteristics from SPICE simulation (left) with the reconstructed ones from MATLAB (right) for $T_1(s)$.

Significant differences between these characteristics may appear if the extraction method described in the previous Section has not been applied correctly. In such cases, the Step 1 has to be repeated, after carefully checking the testbench and the method used to derive the loop gain frequency characteristics, as well as the tool used to extract its poles and zeros.

3.3. Step 3: Plot the Nyquist contour and asses circuit stability based on the Nyquist criterion

The Nyquist contour of T(s) is plotted by using a MATLAB script that takes in the T(s) expression derived in the previous steps. The Nyquist plots for the circuits shown in Figs. 1, 6, 10 and 14 are shown in the followings: Fig. 22 for $T_1(s)$, Fig. 23 for $T_2(s)$ Fig. 24 for $T_3(s)$ and Fig. 25 for $T_4(s)$.

One notes the close correspondence between the Nyquist plots shown in Figs. 22–25 and their counterparts in Figs. 4, 9, 13 and 17, which were obtained by using the mathematical expressions of $T_1(s)$, $T_2(s)$, $T_3(s)$ and $T_4(s)$, as given by eqs. (1), (2), (3) and (4), respectively. This demonstrates the methodology proposed here.

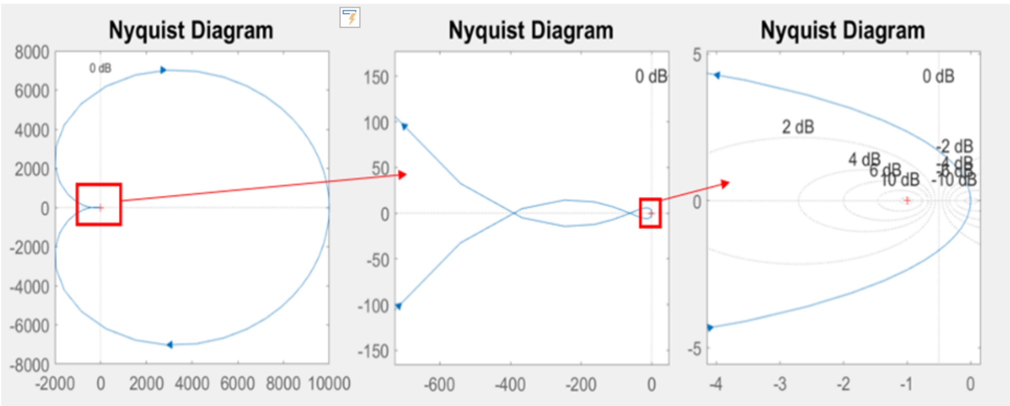

Fig. 22. Nyquist plots yielded by the proposed method for the circuit shown in Fig. 1.

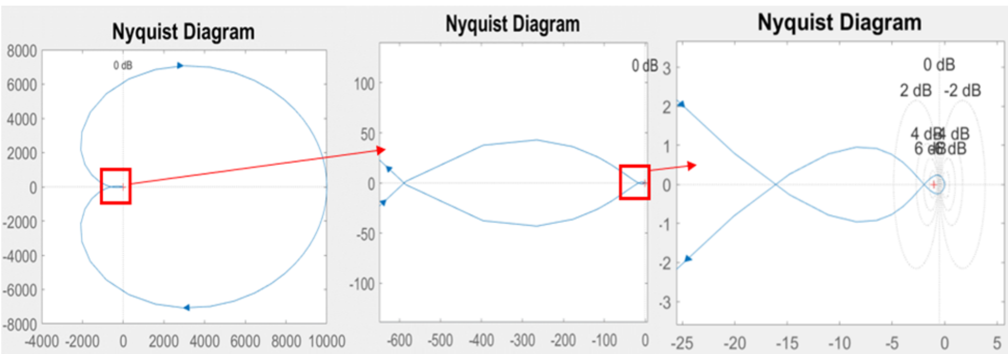

Fig. 23. Nyquist plots yielded by the proposed method for the circuit shown in Fig. 6.

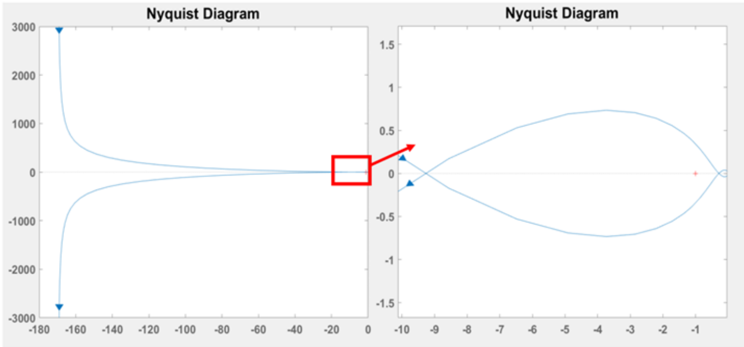

Fig. 24. Nyquist plots yielded by the proposed method for the circuit shown in Fig. 10.

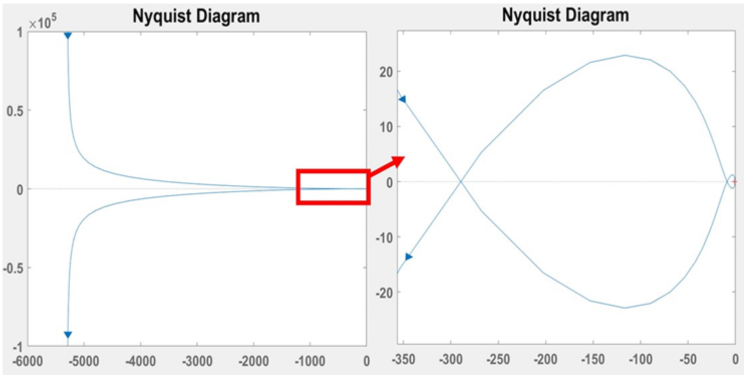

Fig. 25. Nyquist plots yielded by the proposed method for the circuit shown in Fig. 14.

4. Summary and Conclusions

Assessing the stability of feedback systems by using small-signal analysis is time-effective and provides insight into possible design optimization. The Bode stability criterion is the most popular choice and has been implemented in stability analyses provided by widely used simulators. However, the Bode stability criterion is not suitable for assessing the stability of feedback systems for which the phase characteristic of their loop gain crosses the –180° horizontal more than once. Some of these conditionally stable circuits are of certain practical interest, as they allow for a more convenient positioning of poles and zeros than the conventional absolutely stable approach.

Four conditionally stable circuits have been analyzed in some detail to highlight the limitations of the Bode criterion and to illustrate proposal for overcoming these limitations. The loop gain, T(s), of the first two examples comprised no RHP poles, nor poles in the origin. An extension of the Bode criterion has been proposed for such circuits, that allows to assess their stability by analyzing the phase characteristic of their loop gain: if the loop gain phase characteristic crosses the –180° horizontal line an even/odd number of times before the unity gain frequency, the closed loop system is stable/unstable. Furthermore, the closed loop behavior of these circuits was analyzed, in order to derive the relationship between their phase margin and the main parameters of their frequency and step responses: overshoot and gain peaking.

The last two examples of conditionally stable circuits analyzed in this paper have loop gains that comprise poles in the origin and in the RHP. For these circuits it was concluded that a positive phase margin is necessary, albeit it may not be sufficient.

In the third part of this paper, a three-step method for assessing the stability of any conditionally stable system was developed. The poles and zeros of the circuit loop gain are extracted from the frequency characteristics yielded by small signal simulations; a custom MATLAB script uses the frequency characteristics singularities for plotting the Nyquist contour of the circuit. The proposed method was applied to assess the stability of the four conditionally stable circuits analyzed in this paper. The results were validated by analyzing the step response of the circuits.

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