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Problem Setting for Trajectory Planning and Cruise Control of a Connected Autonomous Electric Bus in Intersection Scenarios with Human-Driven Vehicles to Optimize Energy, Comfort and Tracking

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Abstract. This paper proposes a two-stage design approach to ensure reduced energy consumption, improved passenger comfort, and trajectory tracking in the context of an intersection crossing scenario involving a Connected Autonomous Electric Bus (CAEB) and a set of Human-Driven Vehicles (HDVs). The dynamic models of the CAEB and HDVs are defined considering that the movement is in a straight line and using three lanes of the road. In the first stage of the design approach, the discrete-time trajectory planning of the CAEB is performed, which involves the segmentation of the trip into road segments, and the trajectory planning is performed at the level of road segments in discrete time. A novel formula to calculate the length of a road segment is proposed. In this paper a new complete and transparent model is also proposed to compute the signal of traffic lights. An objective function is defined so that it can be minimized in an appropriate optimization problem, which aims to reduce the energy consumption of the CAEB and improve passenger comfort by forcing the CAEB to reduce the frequency of the increments of lane changing. The trajectory planning generates three reference inputs that are used by the cruise controllers designed in the second stage, where the original and simple trigonometric position and speed profiles are defined. In this stage of the design approach, the controllers are designed to ensure trajectory tracking in the framework of a continuous-time optimization problem that aims to reduce energy consumption and improve passenger comfort by minimizing an objective function that considers energy consumption and tracking performance of the CAEB cruise control system. Open-loop simulation results are presented, compared, and discussed for four simulation scenarios.

Key-words: Electric vehicles; energy consumption; human-driven vehicles; optimization; trajectory planning; trajectory tracking.

1. Introduction

According to [1], the development of multimodal transport systems, including connected electric public transport and shared micro-mobility, is necessary to achieve viable, more efficient and equitable mobility services with improved urban mobility and resilience. A partnership involving leading academic institutions, public authorities and industry partners from Romania, Sweden and China has been established to jointly improve next generation multimodal transport systems through electrification, connectivity and sharing. The authors of this paper are affiliated with two of the partners.

In multimodal transportation systems with integrated electric public transport and shared micro-mobility, electric vehicles in public transport are crucial to minimize energy consumption, increase passenger comfort, and optimize operational speed. This can be achieved through strategic infrastructure planning, tactical system optimization, network design and management, and operational vehicle/platoon control and battery management, taking into account different user requirements and behavioral responses. Illustrative examples include overtaking-enabled eco-approach control at signalized intersections for connected and automated vehicles [2], flexible eco-cruising strategy for connected and automated vehicles with efficient driving lane planning and speed optimization [3], and predictive energy-efficient driving strategy design for connected electric vehicles between multiple signalized intersections [4].

Various eco-driving strategies and trajectory planning for connected and automated vehicles have significantly improved efficiency, safety, and traffic flow. Various approaches have been developed to optimize the increment of lane-changing and speed control while minimizing fuel consumption and pollutant emissions. Therefore, to ensure economical travel at signalized intersections for connected and automated vehicles, several modeling, optimization, and control approaches have been proposed in the literature. The literature review on modeling, optimization, and control of connected and automated vehicles at signalized intersections, together with the motivation and the main contributions of this work, are comprehensively detailed in [5]. Building upon the trajectory planning approach introduced in [6], with a different objective function and a similar set of constraints, the current paper proposes a novel two-stage design approach aimed at minimizing energy consumption, enhancing passenger comfort, and improving trajectory tracking for a Connected and Automated Electric Bus (CAEB) in an intersection crossing scenario that includes a set of Human-Driven Vehicles (HDVs).

This paper is organized as follows: Section 2 describes the intersection scenario. The vehicle models are described in detail in Section 3. The proposed two-stage design approach, accompanied by the complete formulations of the optimization problems, is presented in Section 4. Section 5 treats, compares, and discusses the simulation results of the system behavior in four simulation scenarios. The conclusions are highlighted in Section 6.

2. Intersection Crossing Scenario

This paper focuses on a three-lane intersection with fixed signal phase timing, featuring a CAEB and multiple HDVs. The objective is to propose a design approach for the CAEB, aiming at low energy consumption and high passenger comfort. When the design approach is next applied, it will ensure high scheduling reliability. This scenario is shown in Fig. S1 in [5], where overtaking maneuvers are allowed in the context of [2], [3] and [4].

The CAEB is equipped with a Vehicle-to-Infrastructure (V2I) communication device (4G or LTE-V), thus the traffic information (i.e., route distance, traffic signal phase and timing, and speed limits) can be accessed by communicating with road side units or the cloud [3]. The three-lane urban route with a fixed-timing traffic light shown in Fig. S2 (given in [5]) is defined as the set O_r [2]

$$O_r = \{S, D_{C_z}, D_{l_z}, N_l, D_\omega, v_{\text{max}}, v_{\text{min}}\} \tag{1}$$

where S is the location of the stop line at the intersection, also the traffic light position, the road length and the destination position, $S = 300$ m in this problem, D_{C_z} is the length of the communication zone, $D_{l_z} < D_{C_z}$ is the length of the lane-changing coordinator zone, which is related to D_{C_z} and the length of no lane-changing zone near the intersection stop line, N_l is the total number of lanes in the same driving direction as CAEB and HDVs, N_l = 3 in this problem. The numbers of lane from the outside to the inside of the road are $i = 1 \ldots N_l$, different to [3] but in the same style as in [4], D_ω is the width of the lane, $D_\omega = 3.75$ m in this problem, and v_{max} and v_{min} are the maximum and minimum speed limits of CAEB, respectively. As specified in [3], the center of lane 2 ($i = 2$) is defined as the zero lateral position, so the centers of lanes 1 ($i = 1$) and 3 ($i = 3$) are negative and positive, respectively. The coordinate system is also shown in Fig. S2 in [5].

The information on the traffic light is defined as the set O_t [2]

$$O_t = \{S, T_s, I_{in}, T_g, T_r\} \tag{2}$$

where T_s is the initial transition time of the traffic light indication when the CAEB is approaching the communication zone, I_{in} is the initial indication of the traffic light with I_{in} = 1 and I_{in} = 0 denoting the green and red signals, respectively, and T_g and T_r are the time interval of green and red signals, respectively. The value of T_g is recommended to achieve an equilibrium value that satisfies two conditions, (i) and (ii): (i) it must be short enough for drivers to react safely to signal changes, and (ii) it must be long enough to clear the average queue during each traffic signal cycle. The value of T_g can also be set by the user to respond to variations in demand during different time periods, such as morning peak, evening peak, day off-peak, night off-peak, and holidays. The traffic light timing is defined by the initial transition time when the CAEB approaches the communication zone, T_s, the initial signal which indicates green or red, I_{in}, and the green time, T_g, and red time, T_r, intervals, respectively. According to [2] and [4], the yellow interval is merged with the red phase to increase driving safety. The selected green and red durations are different from those given in [7], [8] and [9]. Considering that the green signal changes periodically, the standard signal cycle length $T_l = T_g + T_r$ is determined as described in [4]. The specific values chosen for these intervals in this problem are presented and justified in the final scenario description in Section 4 and the travel route with one signalized intersection is illustrated in Fig. S3 in [5].

The set O_s of surrounding vehicles, namely HDVs, is defined as follows as a modified and extended set of that given in [2]:

$$O_s = \{N_{sv}, i, s_{x,ij}, s_{y,ij}, v_{s,ij}, L_{s,ij}\} \tag{3}$$

where N_{sv} is the number of HDVs, with the vehicle serial numbers specified as $j = 1 \ldots N_{sv}$, in the order of proximity to the intersection, $i = 1 \ldots N_l$ is the driving lane of j^{th} HDV, $s_{x,ij}$ is the longitudinal position of j^{th} HDV in the j^{th} lane, $s_{y,ij}$ is the lateral position of j^{th} HDV in the j^{th} lane, $v_{s,ij}$ is the speed of j^{th} HDV in the j^{th} lane, and $L_{s,ij}$ is the body length of j^{th} HDV in the j^{th} lane.

Considering all possible values of the accumulated number of cycles C of the traffic signal O_t at any moment t as defined in (S2) (given in [5]), and the starting times of the green and red signals for the C cycle, defined as $t_{g,C}$ and $t_{r,C}$ in (S3) and (S4) (given in [5]), respectively, as well as the six possible situations involving t, $t_{g,C}$ and $t_{g,C}$ illustrated in Fig. S5 (given in [5]) and detailed in relation (S5) (given in [5]), the novel formula for determining the traffic light indication P at time t is given by $P(t)$:

$$P(t) = \begin{cases} I_{in}, & \text{if } C = 1, \\ 1, & \text{if } C > 1 \text{ and } \begin{cases} t < t_{r,C} < t_{g,C} & (2) \\ t_{r,C} < t_{g,C} \leq t & (3) \\ t_{g,C} \leq t < t_{r,C} & (6) \end{cases} \\ 0, & \text{if } C > 1 \text{ and } \begin{cases} t < t_{g,C} < t_{r,C} & (1) \\ t_{g,C} < t_{r,C} \leq t & (4) \\ t_{r,C} \leq t < t_{g,C} & (5) \end{cases} \end{cases} \tag{4}$$

3. Vehicle Model

The simplified block diagram of a vehicle with an Ackermann steering design is shown in Fig. S6 (given in [5]). Since, as in [10], the daily driving scenario is assumed to be driven with a reasonable margin to the limit of tire adhesion, tire slip is not considered. It is also assumed that the steering is moderate, and the lane change speed is low, so that the vehicle meets the appropriate dynamics and motion geometry constraints.

The vehicle kinematic model with coupled lateral and longitudinal motions used in [11] is adopted for CAEB and expressed as follows [1]:

$$\dot{x} = v_{\text{CAEB}} \cos\gamma, \quad \dot{y} = v_{\text{CAEB}} \sin\gamma, \quad \dot{\gamma} = v_{\text{CAEB}} \tan(\beta_\omega / L) \tag{5}$$

where v_{CAEB} is the speed of the CAEB, γ is the yaw angle, β_ω is the front wheel steering angle, L is the CAEB body length (L = 13 m in this problem), and s_x and s_y are the longitudinal and lateral positions of the CAEB, respectively [2], [3]. The model in (5) is also expressed in [3], but the second differential equation is different. The initial conditions must also be specified in (5).

Since reducing the energy consumption of CAEBs is one of the objectives of this problem, assuming that a CAEB is driven by a centralized electric motor, the vehicle dynamic model is provided in (S6). The vehicle force generated by the motor is defined in (S7). Finally, the battery power P_b is calculated according to (S8) and the motor power is computed according to (S9) in [5]. The remaining parameters and variables of the CAEB model were collected from [10], [11], [12] and [13], and are detailed in Table S1 in [5].

The kinematic model with coupled lateral and longitudinal motions adopted for CAEB and given in (5) is also used for the HDVs and expressed as follows:

$$\dot{s}_{x,ij} = v_{s,ij} \cos \gamma_{s,ij}, \quad \dot{s}_{y,ij} = v_{s,ij} \sin \gamma_{s,ij}, \quad \dot{\gamma}_{s,ij} = v_{s,ij} \tan\left(\beta_{\omega,ij}/L_{s,ij}\right) \tag{6}$$

where $\gamma_{s,ij}$ is the yaw angle of j^{th} HDV in the i^{th} lane, $\beta_{s,ij}$ is the front wheel steering angle of j^{th} HDV in the i^{th} lane, and the rest of the variables and parameters are specified in (3) and defined immediately after. The initial conditions must also be specified in (6). Therefore, considering the given traffic scenario shown in Fig. S1 and Fig. S2, with HDVs moving in straight lines along the lanes and placed vertically in the centers of the lanes, $\gamma_{s,ij} = 0$ and $\beta_{s,ij} = 0$, the model in (6) actually becomes the particular expression of the first of the three differential equations in (6), namely

$$\dot{s}_{x,ij} = v_{s,ij} \tag{7}$$

and the initial condition must be specified.

4. Problem Formulation and Design Approach

This section is dedicated to the formulation of the two optimization problems, organized as optimal control problems that are used in the design approach. The proposed design approach is then described in the last subsection of this section.

4.1. Optimal control problem for cruise control

The driving lane of the CAEB is affected by the speed and position of HDVs. In this context, as shown in [2] and [3], to realize an efficient CAEB control system such that to maneuver the vehicle efficiently passing through the signalized intersection with a minimal driving cost, it is necessary to introduce a lane change rule to permit the CAEB to change the driving lane to the adjacent left or right lanes or to keep the current lane if could exist slower driving vehicles in the adjacent lane in front of the CAEB, which will be blocked by changing the driving lane.

The state vector **x** of the CAEB is defined in terms of

$$\mathbf{x} = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 \end{bmatrix}^T = \begin{bmatrix} s_x & s_y & \gamma & v_{CAEB} \end{bmatrix}^T \tag{8}$$

where T indicates matrix transposition, and x_1, x_2, x_3 and x_4 are the state variables. The control input vector **u** is expressed as follows, as a modified version of the vector considered in [3]:

$$\mathbf{u} = \begin{bmatrix} u_1 & u_2 & u_3 \end{bmatrix}^T = \begin{bmatrix} T_m & \beta_\omega & L_c \end{bmatrix}^T \tag{9}$$

where u_1, u_2 and u_3 are the control inputs.

To tackle the tradeoff to energy consumption and travel time [4] and to eliminate the influence of the quantity dimensions for both, a monetary counterpart normalized objective function of the CAEB energy consumption and travel time is used to calculate the CAEB driving cost. Therefore, the following optimal control problem for cruise control or trajectory tracking is formulated to reduce driving cost, ensure driving safety, and ensure efficient cruise control:

$$\mathbf{u}_{\text{opt}}(t) = \underset{\substack{\mathbf{u}(t)\in\mathcal{U} \\ t\in[T_i,T_f]}}{\arg\min} J(\mathbf{u}(t), \mathbf{x}(t)) = \underset{\substack{\mathbf{u}(t)\in\mathcal{U} \\ t\in[T_i,T_f]}}{\arg\min} \int_{T_i}^{T_f} \left[\zeta_e P_b(t) + \zeta_t(t) + \mathbf{e}^T(t)\mathbf{Q}\mathbf{e}(t)\right] dt \tag{10}$$

where $\mathcal{U}$ is the feasible domain of $\mathbf{u}(t)$, $\mathbf{u}_{\text{opt}}(t)$ is the optimal control input, T_i is the initial time, T_f is the final time at which the CAEB reaches the destination, i.e. the traffic light, and it is variable. The parameter ζ_e and the variable $\zeta_t(t)$ in the objective function defined in (10) convert the energy and time costs into their monetary counterpart, respectively, based on electricity bill and hourly pay considerations. In the objective function defined in (10), the role of the first term is to enhance vehicle energy efficiency by reducing battery energy consumption, and the second term is defined as cost to improve vehicle mobility. The optimization problem in (10) is subject to several constraints as (S10) and (S18) in [5], which include initial and final conditions, constraint specifying the initial state. Further details on these constraints, safety requirements, and corresponding safety measures are provided in (S11) - (S17) in [5].

4.2. Optimal control problem for trajectory planning to ensure efficient driving lane planning

This subsection is dedicated to trajectory planning, i.e. finding the optimal reference inputs or the optimal desired values or the optimal set-points for the cruise control system. During the whole trip, there are numerous feasible driving lane sequences. A tree graph is illustrated in Fig. S7 (given in [5]) to represent how the CAEB changes lanes while driving with minimum driving cost. The entire trip is segmented into N_s road segments in the spatial domain, where each road segment is defined as a node, with the index $k = 1 \dots N_s$, and the length D_s. Each segment necessitates choosing whether to change lanes or stay in the same lane. It is assumed, as in [2], that the lane change operation of the CAEB continues once it begins until it enters the target lane. As specified in [3], the length D_s of each segment is calculated using the lane change trajectory and the average traffic flow speed, but its calculation will be given as follows using a different approach. As shown in Fig. S6 (given in [5]), D_s is considered on the longitudinal axis s_x in terms of the equidistant positions of the vehicle along successive road segments. The state vector at k^{th} and $(k+1)^{\text{th}}$ segments is expressed in [5]. Using the notations $T_c^{[k]}$ for the travel time of k^{th} segment and $u_3^{[k]} = T_c^{[k]}$ for the control input, where $L_c^{[k]}$ is the lane-changing index of k^{th} segment, the driving lane optimization problem is defined as follows in terms of the combination of the objective functions given in [2] and [3], but simplified. The optimization problem that ensures increment of lane changing reduction and energy reduction is detailed in (S21) in [5].

The energy consumption $E_c^{[k]}$ of k^{th} segment is calculated as follows:

$$E_c^{[k]} = P_b^{[k]} T_c^{[k]} \tag{11}$$

where $P_b^{[k]}$ is the battery power of k^{th} segment, assumed to be constant across all segments, $P_b^{[k]} = P_b$, $k = 1 \dots N_s$. Using (11) in (S21) and considering that the variable $\zeta_t^{[k]}$ is constant across all segments, $\zeta_t^{[k]} = \zeta_t$, $k = 1 \dots N_s$, the optimization problem, which ensures a reduction in the increment of lane change and energy consumption, is transformed into (12) and (13):

$$\begin{aligned} \begin{bmatrix} U_{\text{opt}} \\ V_{\text{opt}} \\ (\gamma_{rc})_{\text{opt}} \end{bmatrix} = \arg \min_{\substack{u_3^{[k]}, v_{CAEBr}^{[k]} \\ k=1\dots N_s \\ \gamma_{rc}}} & (\zeta_t + \zeta_e P_b) T_c^{[N_s]} \\ & + \sum_{k=1}^{N_s - 1} \left[(\zeta_t + \zeta_e P_b) T_c^{[k]} + \zeta \left(u_3^{[k+1]} - u_3^{[k]} \right)^2 \right] \end{aligned} \tag{12}$$

$$
\begin{aligned}
&\begin{bmatrix} U_{\text{opt}} \\ V_{\text{opt}} \\ (\gamma_{rc})_{\text{opt}} \end{bmatrix} = (\zeta_t + \zeta_e P_b) \arg \min_{\substack{u_3^{[k]}, v_{CAEBr}^{[k]} \\ k=1 \ldots N_s \\ \gamma_{rc}}} T_c^{[N_s]} + \sum_{k=1}^{N_s-1} [T_c^{[k]} + (\zeta/(\zeta_t + \zeta_e P_b)) \\
&\cdot (u_3^{[k+1]} - u_3^{[k]})^2] = \arg \min_{\substack{u_3^{[k]}, v_{CAEBr}^{[k]} \\ k=1 \ldots N_s \\ \gamma_{rc}}} T_c^{[N_s]} + \sum_{k=1}^{N_s-1} \left[T_c^{[k]} + \psi \left(u_3^{[k+1]} - u_3^{[k]} \right)^2 \right] \\
&= \arg \min_{\substack{u_3^{[k]}, v_{CAEBr}^{[k]} \\ k=1 \ldots N_s \\ \gamma_{rc}}} J_{DT}(u_3^{[1]}, \ldots, v_{CAEBr}^{[1]}, \ldots, u_3^{[N_s]}, \ldots, v_{CAEBr}^{[N_s]}) \\
&= \arg \min_{\substack{u_3^{[k]}, v_{CAEBr}^{[k]} \\ k=1 \ldots N_s \\ \gamma_{rc}}} [J_{DT1}(u_3^{[1]}, \ldots, v_{CAEBr}^{[1]}, \ldots, u_3^{[N_s]}, \ldots, v_{CAEBr}^{[N_s]}) \\
&+ J_{DT2}(u_3^{[1]}, \ldots, v_{CAEBr}^{[1]}, \ldots, u_3^{[N_s]}, \ldots, v_{CAEBr}^{[N_s]})]
\end{aligned} \tag{13}
$$

subject to the following constraints, including the initial and final conditions:

$$
\begin{aligned}
&u_3^{[k+1]} \in \begin{cases} \{0, 1\}, & \text{if } i^{[k]} = 1 \\ \{-1, 0, 1\}, & \text{if } i^{[k]} = 2 \\ \{-1, 0\}, & \text{if } i^{[k]} = 3 \end{cases} \\
&\left| u_3^{[k+1]} - u_3^{[k]} \right| \le 1, \quad k = 1 \ldots N_s - 1 \\
&t^{[1]} = T_i = 0, \quad t^{[k+1]} = t^{[k]} + T_c^{[k]}, \quad k = 1 \ldots N_s - 1 \\
&s_{xr}^{[1]} = 0, \quad s_{xr}^{[k+1]} = k D_s, \quad k = 1 \ldots N_s - 2, \\
&s_{xr}^{[N_s]} = \begin{cases} S - S_{v,S}, & \text{if } P^{[N_s]} = 0 \\ S, & \text{if } P^{[N_s]} = 1 \end{cases} \\
&s_{yr}^{[1]} = -D_\omega, \quad s_{yr}^{[k+1]} = s_{yr}^{[k]} + D_\omega \cdot \text{sgn}(u_3^{[k]}), k = 1 \ldots N_s - 2 \\
&s_{yr}^{[N_s]} \in \{-D_\omega, 0, D_\omega\}, \\
&\begin{cases} d_{v,ij}^{[k]} - d_{s,ij}^{[k]} \ge 0 \\ d_{v,ij}^{[k]} - d_{s,ij}^{[k]} < 0 \quad \text{and} \quad \begin{cases} u_3^{[k]} = 0 \quad \text{and} \quad \begin{cases} |s_{xr}^{[k]} - s_{x,ij}^{[k]}| \ge H_s + v_{CAEB_r}^{[k]} T_h, \\ \quad k = 1 \ldots N_s \\ v_{CAEB_r}^{[k]} < v_{CAEB_r}^{[k-1]}, \quad k = 2 \ldots N_s \end{cases} \\ u_3^{[k]} \ne 0 \quad \text{and} \quad |s_{yr}^{[k]} - s_{y,ij}^{[k]}| \ge H_s \\ |s_{xr}^{[k]} - s_{x,ij}^{[k]}| \ge H_s, \quad k = 1 \ldots N_s \end{cases} \\ i = 1 \ldots 3, \quad j = 1 \ldots N_{sv} \end{cases} \\
&v_{CAEB_{\min}} \le v_{CAEB_r}^{[k]} \le v_{CAEB_{\max}}, \quad \tan \gamma_{rc} > 4.5 D_\omega / D_{c_z} \\
&u_3^{[1]} = 0, \quad u_3^{[N_s]} = 0
\end{aligned} \tag{14}
$$

Explanations of the last two pairs of constraints in (14) are provided in the context of (S29) in [5]. The overall objective function J_{DT} depends on the decision variables of the optimization problem, $\varsigma > 0$ is the normalized coefficient for the penalty of frequent lane change increements, $\psi > 0$ is the weight of the frequent lane changes versus the monetary counterpart of the time cost defined in (S74) in [5], U_{opt} is the optimal sequence of control inputs:

$$U_{\text{opt}} = \left\{ \left(u_3^{[k]} \right)_{\text{opt}} \middle| k = 1 \ldots N_s \right\} = \left\{ \left(L_c^{[k]} \right)_{\text{opt}} \middle| k = 1 \ldots N_s \right\} \tag{15}$$

$(u_3^{[k]})_{opt} = (T_c^{[k]})_{opt}$ is the optimal control input of k^{th} segment, V_{opt} is the optimal sequence of CAEB speeds:

$$V_{\text{opt}} = \left\{ \left(v_{CAEBr}^{[k]} \right)_{\text{opt}} \middle| k = 1 \ldots N_s \right\} \tag{16}$$

$(v_{CAEBr}^{[k]})_{\text{opt}}$ is the optimal CAEB speed along k^{th} segment, and $(\gamma_{rc})_{\text{opt}}$ is the optimal value of the desired yaw angle γ_{rc} of the CAEB at the center of the road segment, which is used in [5] to derive the segment length and subsequently determine the travel time for lane changes between segments. The value of $(\gamma_{rc})_{\text{opt}}$ is constant over all road segments, $k = 1 \ldots N_s$.

The weight parameter $\zeta_e > 0$ (assumed to be constant over a given segment) and the variable $\zeta_t^{[k]} > 0$ (assumed to be constant over a given segment) are described in relation with (12), (13) and (S21) in [5]. The two components of the discrete time objective function J_{DT}, i.e. J_{DT1} and J_{DT2}, are detailed in (S75) in [5].

The mathematical formulation and the derivation of the constraints in (14) are comprehensively detailed in (S23) - (S57), (S62) - (S68), (S70) - (S73) in [5].

Once the optimization problem in (12) is solved accounting for the constraints in (14), the CAEB efficient-driving trajectory is defined in terms of the following sets:

$$\begin{aligned} T_{s_{xr},\text{opt}} &= \left\{ \left(s_{xr}^{[k+1]} \right)_{\text{opt}} \middle| k = 1 \ldots N_s - 2 \right\} \cup \left\{ s_{xr}^{[N_s]} \right\} \\ &= \begin{cases} \{kD_s \,|\, k = 1 \ldots N_s - 2\} \cup \{S - S_{v,S}\}, & \text{if } P^{[N_s]} = 0 \\ \{kD_s \,|\, k = 1 \ldots N_s - 2\} \cup \{S\}, & \text{if } P^{[N_s]} = 1 \end{cases} \end{aligned} \tag{17}$$

$$\begin{aligned} T_{s_{yr},\text{opt}} &= \left\{ \left(s_{yr}^{[k+1]} \right)_{\text{opt}} \middle| k = 1 \ldots N_s - 2 \right\} \cup \left\{ s_{yr}^{[N_s]} \right\} \\ &= \{-D_\omega\} \cup \left\{ \left(s_{yr}^{[k+1]} \right)_{\text{opt}} \middle| k = 2 \ldots N_s - 2 \right\} \cup \{-D_\omega, 0, D_\omega\} \end{aligned} \tag{18}$$

and the set V_{opt} defined in (16), where $T_{s_{xr},\text{opt}}$ and $T_{s_{yr},\text{opt}}$ are the optimal longitudinal trajectory and the optimal lateral position trajectory of the CAEB, respectively, and $(s_{xr}^{[k]})_{\text{opt}}$ and $(s_{yr}^{[k]})_{\text{opt}}$ are the optimal longitudinal position and the optimal lateral position of the CAEB speed along k^{th} segment, respectively.

The original expression of D_s proposed in this paper and explained in [5] depends only on γ_{rc} as shown in (S49) in [5]. When the CAEB is driven at low speed to ensure comfort, the value of γ_{rc} is low and the value of D_s is high. On the other hand, when the CAEB is operated with a "sporty" approach, the magnitude of γ_{rc} is high, resulting in a low value of D_s. After computing D_s, the number N_s of road segments is determined according to (S51) in [5], with D_{C_z} specified in (1).

In the lane-keeping maneuver, as considered in [3], the trigonometric speed profile is applied to smoothly connect the CAEB speed between two consecutive road segments. The following original speed profile is proposed in this paper and derived in [5], as a simplified version of the profiles considered in [3] and [14] in terms of fewer parameters and elegant mathematical manipulation, inspired by the trigonometric membership functions of fuzzy sets used in [15]–[17]:

$$v_{CAEBr}(t) = v_{CAEBr}^{[k]} + \frac{v_{CAEBr}^{[k+1]} - v_{CAEBr}^{[k]}}{2} \cdot \left[1 + \cos\left(\frac{\pi(t - t^{[k]} - T_c^{[k]})}{T_c^{[k]}}\right)\right], \quad t \in [t^{[k]}, t^{[k]} + T_c^{[k]} = t^{[k+1]}) \tag{19}$$

Using the notations in (S58) in [5], the mathematical formulation and derivation in (S59) - (S61) and (S69) in [5], the expression of the travel time $T_c^{[k]}$ of k^{th} segment during both lane-changing and lane-keeping is

$$T_c^{[k]} = \begin{cases} \dfrac{2D_s}{v_{CAEBr}^{[k+1]} + v_{CAEBr}^{[k]}}, \text{if } k = 1 \ldots N_s - 1 \text{ and } u_3^{[k]} = 0 \\ \dfrac{D_s}{v_{CAEBr}^{[k]}}, \quad \text{if } k = N_s \text{ and } u_3^{[k]} = 0 \\ \dfrac{f}{v_{CAEBr}^{[k]}}, \quad \text{if } k = 1 \ldots N_s \text{ and } u_3^{[k]} \neq 0 \end{cases} \tag{20}$$

Finally, the start time set $T_{s,\text{opt}}$ and the final time set $T_{f,\text{opt}}$ of all segments are derived from the travel time of each road segment as follows:

$$T_{s,\text{opt}} = \{t^{[1]}, t^{[2]}, \ldots t^{[N_s]}\} = \{T_i = 0, T_c^{[1]}, \sum_{k=1}^{2} T_c^{[k]}, \ldots, \sum_{k=1}^{N_s-1} T_c^{[k]}\} \tag{21}$$

$$\begin{aligned} T_{f,\text{opt}} &= \{t^{[1]} + T_c^{[1]}, t^{[2]} + T_c^{[2]}, \ldots T_f = t^{[N_s]} + T_c^{[N_s]}\} \\ &= \{T_c^{[1]}, \sum_{k=1}^{2} T_c^{[k]}, \ldots, T_f = \sum_{k=1}^{N_s} T_c^{[k]}\} \end{aligned} \tag{22}$$

The remaining parameters and variables of the CAEB model are collected from [2], [3] and [18], and are detailed in Table S2 given in [5].

To summarize the results presented in this section and in the previous sections, several parameters are set regarding the CAEB. These parameters are described in [5].

4.3. Optimal speed control approach

Summarizing the description of the optimization problems defined and described in the previous subsections, the optimal speed control approach for the CAEB consists of two stages, I and II, which are specified as follows after synthesizing the results given in the previous two subsections.

Stage I. This is the optimal trajectory planning stage. The optimization problem defined in (12) and (13) is solved, subject to the constraints in (14), the final condition in (S76) and the initial condition in (S77) in [5], resulting in the optimal sequence of lane change indices U_{opt} in (14), and the optimal sequence of CAEB speeds V_{opt} in (15), each consisting of N_s elements.

The initial conditions specified above for the optimization problem in (12) determine the dynamic regime in which the objective function J_{DT} defined in (S75) is evaluated.

The optimal longitudinal trajectory $T_{s_{xr},\text{opt}}$ and the optimal lateral position trajectory $T_{s_{yr},\text{opt}}$ are calculated using (21) and (22), respectively.

Stage II. This is the optimal cruise control stage. The three cruise control reference inputs are calculated on continuous time horizons to allow for continuous time control. Assuming a linear variation of the desired longitudinal position of the CAEB, s_{xr}, over the k^{th} road segment, the value of s_{xr} at the continuous time instant t is calculated as $s_{xr}(t)$:

$$s_{xr}(t) = \begin{cases} s_{xr}^{[k]} + \dfrac{(s_{xr}^{[k+1]} - s_{xr}^{[k]})(t - t^{[k]})}{T_c^{[k]}} = (k-1)D_s + \dfrac{(t - t^{[k]})D_s}{T_c^{[k]}}, \\ \text{if } k = 1 \ldots N_s - 1 \\ s_{xr}^{[k]}, \quad \text{if } k = N_s \end{cases}$$
$$t \in [t^{[k]}, t^{[k]} + T_c^{[k]} = t^{[k+1]}) \tag{23}$$

Assuming a trigonometric variation of the desired lateral position of the CAEB, s_{yr}, over the k^{th} road segment, similar to the trigonometric speed profile given in (19) in order to smoothly connect the CAEB lateral position between two consecutive road segments, the value of s_{yr} at the continuous time instant t is calculated as $s_{yr}(t)$:

$$s_{yr}(t) = \begin{cases} s_{yr}^{[k]} + \dfrac{s_{yr}^{[k+1]} - s_{yr}^{[k]}}{2}\left[1 + \cos\dfrac{\pi(t - t^{[k]} - T_c^{[k]})}{T_c^{[k]}}\right], \text{if } k = 1 \ldots N_s - 1 \\ s_{yr}^{[k]}, \quad \text{if } k = N_s \end{cases}$$
$$t \in [t^{[k]}, t^{[k]} + T_c^{[k]} = t^{[k+1]}) \tag{24}$$

Using the original trigonometric speed profile in (19) in the lane-keeping maneuver and keeping the constant speed in the lane-changing maneuver, the expression of the reference input v_{CAEBr} or the set-point for the speed of the CAEB or the desired speed of the CAEB at the continuous time instant t over the k^{th} road segment is calculated as $v_{CAEBr}(t)$:

$$v_{CAEBr}(t) = \begin{cases} v_{CAEBr}^{[k]} + \dfrac{v_{CAEBr}^{[k+1]} - v_{CAEBr}^{[k]}}{2}\left[1 + \cos\dfrac{\pi(t - t^{[k]} - T_c^{[k]})}{T_c^{[k]}}\right], \\ \text{if } k = 1 \ldots N_s - 1 \\ v_{CAEBr}^{[k]}, \text{if } k = N_s \end{cases}$$
$$t \in [t^{[k]}, t^{[k]} + T_c^{[k]} = t^{[k+1]}) \tag{25}$$

The optimal control problem defined in (10) is solved, with **u** in (S78) in [5] and subject to the constraints given in (26) and obtained from (S18) by dropping out the constraint imposed to the lane-changing index, and adding the last three constraints, which model the reference inputs. The cruise controller obtained by solving (10) subject to (26) ensures a trade-off to energy cost and tracking accuracy. Certain controller structures can be considered with appropriate parameterization, turning the optimization problem (10) into a parametric optimization problem.

$$
\begin{aligned}
&\dot{\mathbf{x}}(t)=\mathbf{f}(\mathbf{x}(t),\mathbf{u}(t)) \text{ according to (S19)}\\
&v_{CAEB_{\min}} \le x_4(t) \le v_{CAEB_{\max}}\\
&a_{\min} \le \frac{[u_1(t)i_g i_0 \eta_t^{\operatorname{sgn}(u_1(t))}/r_w - (mgf\cos\theta + mg\sin\theta + 0.5C_D A\rho x_4^2(t))]}{m\delta} \le a_{\max}\\
&T_{m\min} \le u_1(t) \le T_{m\max}\\
&\begin{cases}
d_{v,ij}(t)-d_{s,ij}(t)\ge 0\\
d_{v,ij}(t)-d_{s,ij}(t)< 0 \text{ and }
\begin{cases}
u_3(t)=0 \text{ and }
\begin{cases}
|s_x(t)-s_{x,ij}(t)| \ge H_s\\
+ v_{CAEB}(t)\cdot T_h,\\
([u_1(t)i_g i_0 \eta_t^{\operatorname{sgn}(u_1(t))}/r_w\\
-(mgf\cos\theta+mg\sin\theta\\
+0.5C_D A\rho x_4^2(t))])/(m\delta)<0,
\end{cases}\\
u_3(t)\ne 0 \text{ and } |s_y(t)-s_{y,ij}(t)|\ge H_s,\\
|s_x(t)-s_{x,ij}(t)|\ge H_s,
\end{cases}
\end{cases}\\
&i=1\ldots 3,\quad j=1\ldots N_{sv}\\
&\begin{cases}
v_{CAEB}(T_f)=0, \text{ and } s_x(T_f)=S-S_{v,S}, \text{ and } \dot{s_y}(T_f)=0, & \text{if } P(T_f)=0\\
v_{CAEB}(T_f)\ne 0, \text{ and } s_x(T_f)=S, \text{ and } \dot{s_y}(T_f)=0, & \text{if } P(T_f)=1
\end{cases}\\
&x_1(T_i)=0,\quad x_2(T_i)=-D_\omega,\quad x_3(T_i)=0,\quad x_4(T_i)=v_s\\
&x_1(T_f)=\begin{cases} S-S_{v,S}, & \text{if } P(T_f)=0\\ S, & \text{if } P(T_f)=1\end{cases},\quad x_2(T_f)\in\{-D_\omega,0,D_\omega\},\quad x_3(T_f)=0\\
&s_{xr}(t) \text{ according to (23)},\quad s_{yr}(t) \text{ according to (24)},\quad v_{CAEBr}(t) \text{ according to (25)}
\end{aligned}
\tag{26}
$$

The two optimization problems in stages I and II are not solved in this phase, i.e., optimal trajectory planning and optimal cruise control are not performed. In other words, the process behavior is considered with random control inputs in the trajectory planning and open-loop cruise control.

5. Simulation of System Behavior and Discussion of Simulation Results

The traffic system is simulated using a real urban route as shown in Fig. S1 given in [5]. The system under consideration involves a CAEB, whose kinematic model given is given in [11] to predict the behavior of the CAEB under different dynamic regimes, and six HDVs, with the notations specified in Fig. S1 and Fig. S2 given in [5]. The primary objective of this system is to ensure autonomous and efficient control, which includes energy management, obstacle avoidance, and trajectory tracking. However, as mentioned above, this paper considers the open-loop system as far as both trajectory planning and trajectory tracking are concerned. The parameters of the intersection scenario and the four simulation scenarios are presented in the next subsection.

5.1. Intersection scenario parameters and simulation scenarios

The simulation of the traffic system and the associated traffic flow in representative cases was performed in Matlab & Simulink with details given in [5]. The values of all parameters involved in the traffic system model are given in Table S1 and Table S2 in [5], and only the values of the variable parameters in these two tables are given below as they are used in the simulation of the traffic system. The coefficients that convert energy consumption and travel time into their monetary counterparts are specified in Table 2 based on $\zeta_e = 0.12$ USD/kWh and $\zeta_t = 24$ USD/h. The normalized coefficient for the penalty of frequent increments of lane changes is $\varsigma = 0.03$ USD, which is determined by analyzing the average number of increment of lane changing. The parameters and initial conditions for HDVs are detailed in [5]. In this application, the fixed parameters of the traffic light in (2) are set to: the initial transition time of the traffic light indication when the CAEB is approaching the communication zone is T_s = 1 s, the green signal duration is T_g = 9 s, and the red signal duration is T_r = 5 s. These values deviate from the standard durations provided in [10], [11] and [13]. Additionally, as specified in relation with (2), I_{in} is the initial indication of the traffic light with I_{in} = 1 and I_{in} = 0 denoting the green and red signals, respectively.

Two different values are assigned to the desired yaw angle γ_{rc} to generate two simulation scenarios. The first imposed value of the desired yaw angle is set to $\gamma_{rc} = 15^o$ and the second desired yaw angle is set to $\gamma_{rc} = 20^o$. These variations in the desired yaw angle influence the road segment length D_s and the number of segments N_s resulting in different configurations for each simulation scenario. Additionally, the two possible values of the initial indication of the traffic light are considered, namely I_{in} = 1 and I_{in} = 0, leading to other two simulation scenarios. The combination of these two values of γ_{rc} and two values of I_{in} results in four simulation scenarios 1 to 4, which are defined as follows: *Scenario 1.* This scenario is characterized by $\gamma_{rc} = 15^o$ and I_{in} = 1. *Scenario 2.* This scenario is characterized by $\gamma_{rc} = 15^o$ and I_{in} = 0. *Scenario 3.* This scenario is characterized by $\gamma_{rc} = 20^o$ and I_{in} = 1. *Scenario 4.* This scenario is characterized by $\gamma_{rc} = 20^o$ and I_{in} = 0. The graphical analysis of the system responses in these four simulation scenarios is presented in the next subsection. Several figures are included and discussed.

Each scenario is discussed by a separate set of 9 figures that provide insight into the system behavior under different traffic scenarios: Figs. S9 through S16 for scenario 1, Figs. S17 through S24 for scenario 2, Figs. S25 through S32 for scenario 3, and Figs. S33 through S40 for scenario 4. The figures along with discussions are given in [5].

6. Conclusions

This paper introduced a novel two-stage design approach to ensure reduced energy consumption, improved passenger comfort, and trajectory tracking in the context of an intersection crossing scenario involving a Connected Autonomous Electric Bus (CAEB) and a set of Human-Driven Vehicles (HDVs). The two-stage approach consists of CAEB trajectory planning (stage I) and CAEB trajectory tracking (stage II), associated with the proper definitions of two optimization problems formulated in discrete time (stage I) and in continuous time (stage II) along with specifying the decision variables and deriving the constraints that account for real traffic conditions.

Besides the design approach, the novelties of the paper are: the objective functions in the two optimization problems, the formula to compute the length of a road segment, the complete and transparent model to compute the traffic light indication, and the definition of simple trigonometric position and speed profiles. All these novelties are important in the context of the state of the art, and are illustrated in the modeling results of four open-loop simulation scenarios for a given intersection scenario. The results of the simulation scenarios show that the proposed approach has a big potential for practical applications. The need to solve the optimization problems and ensure optimal trajectory planning and tracking is shown. That will result in smooth trajectories, lower power consumption, and slightly increased tracking accuracy. The main limitation of the proposed approach is the offline design of the trajectory planning (in stage I), which requires accurate information about the traffic conditions based on appropriate measurement instrumentation and communication devices. However, this is compensated by the hardware and software support of the CAEB and the traffic system in which the intersection is installed.

Future research will focus on solving the two optimization problems involved in the design approach. This will lead to the optimal desired trajectory of the CAEB, formulated at the level of road segments, and the optimal trajectory tracking. Several optimization algorithms and controller structures taken from various applications including active structures [19], tower crane systems [20], maglev trains [21] and mobile robots [22] will be integrated to solve these problems.

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